Panel 1

Last Time

Optimization: f of one variable

$f' = 0 \Rightarrow$ answer

Implicit Diff: $f(x,y) = c$

Related Rates: $A = x^2$, $A = A(t)$, $x = x(t)$

$\frac{d}{dt} A(t) = \frac{d}{dt} x(t)^2 \Rightarrow A' = 2x \cdot x'$

Linearization: $f(x) \approx f'(c) \cdot (x-c) + f(c)$ near $x = c$

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Panel 2

$x^3 + x^2 y + x y^2 + y^3 = 5$

$y = y(x)$: x is variable, y is a function (Find y')

$\frac{d}{dx}\left(x^3 + x^2 y + x y^2 + y^3\right) = \frac{d}{dx}(5)$

$3x^2 \cdot \underbrace{\cancel{\frac{dx}{dx}}}_{1} + 2x \cdot 1 \cdot y + x^2 \cdot y' + y^2 + x \cdot 2y \cdot y' + 3y^2 \cdot y' = 0$

$x = x(y)$, y is var., x is a function (Find x')

$\frac{d}{dy}\left(x^3 + x^2 y + x y^2 + y^3\right) = \frac{d}{dy}(5)$

$3x^2 \cdot x' + 2x \cdot x' y + x^2 \cdot 1 + x' y^2 + x \, 2y + 3y^2 = 0$

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Panel 3

$\sin(x)\cdot\cos(y) = 1$, $x = x(t), y = y(t)$, t var.

$\cos(x)\cdot x'\cdot\cos(y) + \sin(x)\cdot(-\sin(y))\cdot y' = 0$

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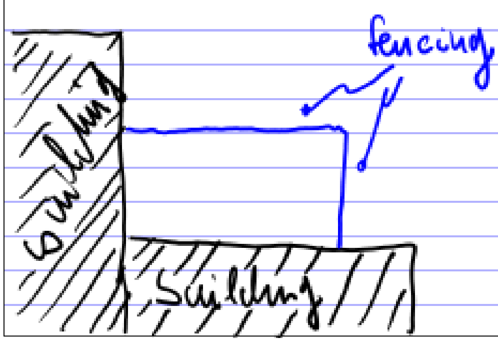
Panel 4

Quiz #9 Name: ________

① Joe wants to fence in a rectangular plot of land for a veggie garden in the corner between two buildings. He has been given 50m of fencing and wants to enclose the max. area. What dimensions should he choose?



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Panel 5

(2) The equation $x^2 + xy + y^2 = 2$ defines y as a function of x. Find dy/dx

(3) The equation $V = \frac{4}{3}\pi r^3$ defines the volume of a ball as a function of r. Assuming both V and r are functions of t, find dV/dt

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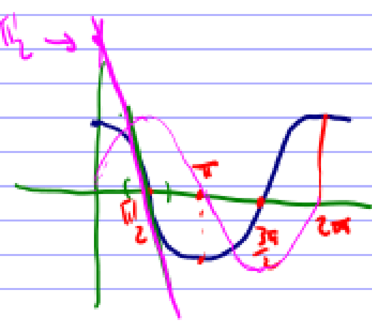
Panel 6

Linearization: $f(x) \approx f'(c)(x-c) + f(c)$

Last time we found the lin. of $\sin(x)$ near 0. Now find the lin. of $\cos(x)$ near $\pi/2$ and use it to approx. $\cos(1.5)$



$\cos(x) \approx -\sin(\frac{\pi}{2})(x - \frac{\pi}{2}) + \cos(\frac{\pi}{2})$

$= -1(x - \pi/2) + 0$

$= \pi/2 - x$

$\frac{3.14}{2}$

$\Rightarrow \cos(1.5) \approx \frac{\pi}{2} - 1.5 = 1.57 - 1.5 = 0.07$

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Panel 7

Differentials

The differential dy is defined as

$$\frac{dy}{dx} = f'(x) \Rightarrow \overset{\text{difference}}{dy} = f'(x)\overset{\text{difference}}{dx}$$

Ex: $f(x) = \sqrt{x+3}$. Find dy if $x=1$ and $dx = 0.05$

$dy = f'(1)\cdot dx$, $f'(x) = \frac{1}{2}(x+3)^{-1/2} \Rightarrow f'(1) = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$

$dy = \frac{1}{4}\,0.05$

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Panel 8

Error Estimation: The radius of a sphere was measured as 21 cm with an error of 0.05 cm. What is the impact of this error if radius is used to compute volume of the sphere.

$V = \frac{4}{3}\pi r^3$

$dV = \frac{4}{3}\pi 3r^2\,dr$

$= 4\pi r^2\,dr$

$dr = 0.05$

$r = 21$

$dV = 4\pi\, 21^2 \cdot 0.05 = 277$ cm

An error of 0.05 cm in the radius results in an error of ± 277 cm³ in volume!

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Panel 9

Relative Error: The relative error is $\frac{df}{f}$

Ex: Find relative error in prev example

$dr = 0.05$, $r = 21$ $\Rightarrow$ rel. error in radius: $\frac{0.05}{21} = 0.0024$

$= 0.24\%$

$dV = \frac{277}{38792} = 0.0071$ or 0.71%

$V = \frac{4}{3}\pi r^3 = 38792$

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Panel 10

Summary of Applications of Derivatives

(1)

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