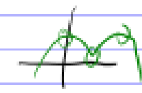
Panel 1

Local Extrema, Inflection Points, and the like



Basic Theorem: If f has a local extrema at $x=c$, then $f'(c)=0$ or $f'(c)$ does not exist.

If $f'(x)>0$: f is increasing $\nearrow$

$f'(x)<0$: f is decreasing $\searrow$

$f''(x)>0$: f is concave up $\cup$

$f''(x)<0$: f is concave down $\cap$

Def: If $f'(c)=0$ or $f'(c)$ d.n.e. $\to$ x is critical pt.

If $f''(c)=0$ or $f''(c)$ d.n.e. $\to$ x is possible infl. pt.

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Panel 2

Curve Sketching

1. Find domain
2. Find y-intercept
3. Find horiz. and vert. asymptotes $\Leftarrow$
4. Find critical pts
5. Find critical points ($f'=0$ or d.n.e.)
6. Find possible inflection point ($f''=0$ or d.n.e.)
7. Create THE TABLE
8. Create THE VALUES
9. Sketch the graph

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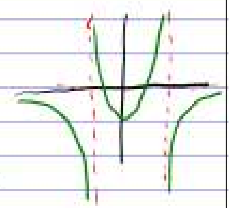
Panel 3

Remember Asymptotes



Horizontal asymp.: if $\lim_{x\to\pm\infty} f(x) = \#$

then $y = \#$ is h.a.

Vertical asymp.: if $\lim_{x\to c} f(x) = \pm\infty$

then $x = c$

Ex: $f(x) = \dfrac{3x^2}{x^2-1}$ $\boxed{y=3 \text{ is h.a., } x=\pm 1 \text{ is v.a.}}$

$$\lim_{x\to\pm\infty} \frac{3x^2}{x^2-1} = 3$$

$x=1$: $\lim_{x\to 1} \dfrac{3x^2}{x^2-1} = \pm\infty$

$x=-1$: $\lim_{x\to -1} \dfrac{3x^2}{x^2-1} = \pm\infty$

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Panel 4

Sketch $f(x) = \dfrac{2x^2}{x^2-1}$

$$f'(x) = \frac{4x(x^2-1) - 2x^2\cdot 2x}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2}$$

$$f''(x) = \frac{-4(x^2-1)^{\cancel{2}} + 4x\cdot 2\cancel{(x^2-1)^1}\cdot 2x}{(x^2-1)^{\cancel{4}\,3}} =$$

$$= \frac{-4x^2+4+16x^2}{(x^2-1)^3} = \frac{12x^2+4}{(x^2-1)^3}$$

1. Domain
2. Intercepts
3. Asymptotes
4. Critical pts
5. poss. infl. pts
6. the Table
7. the Values
8. the Graph

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Panel 5

Sketch $f(x)=\frac{2x^2}{x^2-1}$. Hint: verify

Hint $f'(x)=\frac{-4x}{(x^2-1)^2}$, $f''(x)=\frac{12x^2+4}{(x^2-1)^3}$

1. Domain
2. Intercepts
3. Asymptotes
4. Critical pts
5. poss. infl. pts
6. the Table
7. the Values
8. the Graph

(1) $x \neq \pm 1$

(2) $y=0$ ✓ (set $x=0$)

(3) v.a.: $x=\pm 1$, h.a.: $y=2$

(4) $x=0$, $x=\pm 1$

(5) $x=\pm 1$

(6) $\longrightarrow$

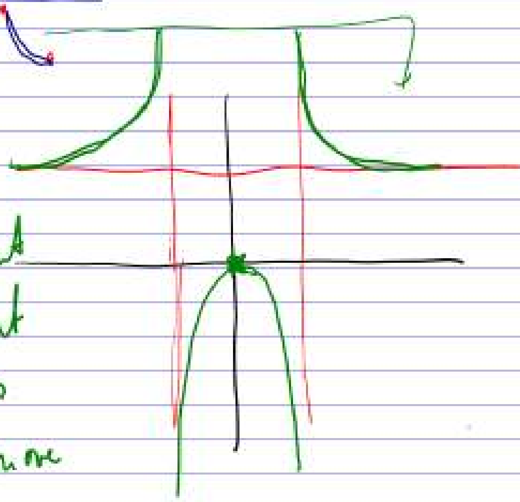
	-1	0	1	
f'	+	+	−	−
f''	+	−	−	+
f				

(7) $(0, f(0))=$
$(0,0)$

Panel 6

	-1	0	1	
f'	+	+	−	−
f''	+	−	−	+
f				

$(0,0)$



Note: you can not cross v.a., but you could cross h.a. once or more

Panel 7

Sketch $f(x)=\frac{x}{x^3-2}$

1. Domain
2. Intercepts
3. Asymptotes
4. Critical pts
5. poss. infl. pts
6. the Table
7. the Values
8. the Graph

$$f' = \frac{1\cdot(x^3-2) - x\cdot 3x^2}{(x^3-2)^2} = \frac{-2x^3-2}{(x^3-2)^2} = \frac{-2(x^3+1)}{(x^3-2)^2}$$

$$f''(x) = \frac{-6x^2(x^3-2)^{\cancel{2}\,1} + 2(x^3+1)\,2\cancel{(x^3-2)^1}\,3x^2}{(x^3-2)^{\cancel{4}\,3}} =$$

$$= \frac{-6x^5+12x^2+12x^5+12x^2}{(x^3-2)^3} =$$

$$= \frac{6x^5+24x^2}{(\quad)^3} = \frac{6x^2(x^3+4)}{(\quad)^3}$$

Panel 8

Sketch $f(x)=\frac{x}{x^3-2} \Rightarrow f'(x) = \frac{-2(x^3+1)}{(x^3-2)^2}$

$$f''(x) = \frac{6x^2(x^3+4)}{(x^3-2)^3}$$

1. Domain
2. Intercepts
3. Asymptotes
4. Critical pts
5. poss. infl. pts
6. the Table
7. the Values
8. the Graph

(1) $x \neq \sqrt[3]{2}$, (2) $(0,0)$

(3) v.a.: $x = \sqrt[3]{2}$

h.a. $y = \lim_{x\to\pm\infty} \frac{x}{x^3-2} = 0$

(4) $x=-1$, $x = \sqrt[3]{2}$

(5) $x=0$, $x=-\sqrt[3]{4}$, $x=\sqrt[3]{2}$

	$-\sqrt[3]{4}$	-1	0	$\sqrt[3]{2}$	
f'	+	+	−	−	−
f''	+	−	−	−	+
f					

$f(0) = 0$

$f(-1) = 1/3 = 0.33$

$f(-\sqrt[3]{4}) = 0.265$

Panel 9

	$-\sqrt[3]{4}$		-1		0		$\sqrt[3]{2}$	
f'	+	+		−		−		−
f''	+	−		−		−		+

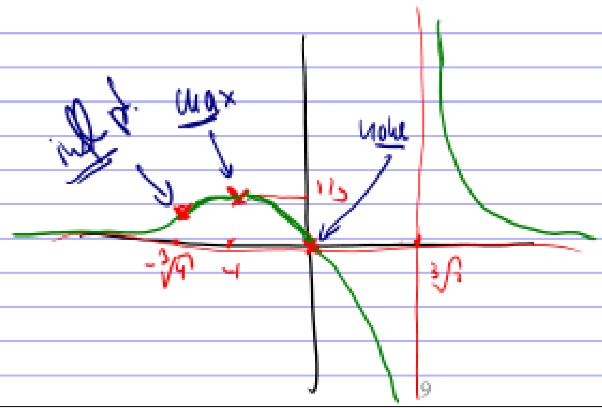
$f(0) = 0$

$f(-1) = \frac{1}{3} = 0.33$

$f(-\sqrt[3]{4}) = 0.265$

h.a. $y = 0$

v.a.: $x = \sqrt[3]{2}$



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Panel 10

So far:

1. Find local max/min, incr/decr.
2. Find infl pts, concave up/down
3. Curve sketching (1 + 2 + asymp)

Next: Finding absolute extrema.

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Panel 11

Theorem: f is cont. on $[a,b]$ ← closed + solid. Then f has abs. max and min. They can occur at critical points or at endpoints.

Ex: $f(x) = x^3 - 3x^2 + 1$ on $[-1/2, 4]$

Find abs. max/min

critical: $f'(x) = 3x^2 - 6x = 3x(x-2)$, $x = 0, 2$ (inside $[-1/2, 4]$)

x	
0	1
2	-3 ⟸ abs. min.
$-1/2$	$1/8$
4	17 ⟸ abs. max

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Panel 12

Ex: $f(x) = \dfrac{x}{x^2+4}$, $x \in [0,3]$. Abs. max/min.

$$f'(x) = \frac{1\cdot(x^2+4) - x\,2x}{(x^2+4)^2} = \frac{-x^2+4}{(x^2+4)^2} = \frac{(2-x)(2+x)}{(x^2+4)^2} = 0$$

critical: $x = +2, -2$

x		
$\cancel{-2}$	not in $[0,3]$	
2	$1/4 = 0.25$	← max
0	0	← min
3	$3/13 \approx 0.23$	

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