Panel 1

Last time:

- Review of local extrema
- Abs. extrema
- Incr/decr
- Concave up/down
- Asymptotes
- Graphing

Various theorems

Panel 2

Rolle’s Theorem: If $f$ is differentiable on $(a,b)$ and continuous on $[a,b]$, and then

Related to Rolle’s theorem is →
Panel 3

**Mean Value Theorem (MVT):**

Let $f$ be a function defined on $(a,b)$, continous on $[a,b]$. Then,

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

The slope of the line $\frac{f(b)-f(a)}{b-a}$.

Example: $f(x)=x^3-x$, $a=0$, $b=2$ 

$$c = \frac{2}{3}$$

Panel 4

**Proof of MVT.**

Define $L(x) = f(x) - \left[ f(a) + \frac{f(b)-f(a)}{b-a} (x-a) \right]$.

$\Rightarrow L'(c) = 0 \Rightarrow L(b) = 0 $

Then, if $f'(x) = 0 \forall x$ on $(a,b)$ then $f$ is constant.

Take any $x,y$, $f'(c) = \frac{f(y)-f(x)}{y-x} = 0$ 

No matter what $x,y$ is!}

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Panel 5

Optimization Problems

A farmer has 2400 ft of fencing to enclose a rectangular plot bordering a river. No fence is needed on river. What dimensions will give the largest area?

Panel 6

A cylindrical can should hold 1 l of oil. Minimize cost of metal used to manufacture the can.

\[ V = \pi r^2 h = 1 \]
\[ l = 2\pi r h + (2\pi r^2) \]

\[ = 2\pi \left( \frac{r}{\pi} \right) + 2\pi r^2 \]
\[ = \frac{2}{r} + 2\pi r^2 \]

\[ A^1 = -\frac{2}{r^2} + 4\pi r = \frac{4\pi r^3 - 2}{r^2} = 0 \Rightarrow r^3 = \frac{1}{2\pi} \Rightarrow r = \sqrt[3]{\frac{1}{2\pi}} \]

Why is it a minimum?
Panel 7

A man launches a boat from point A on a river bank, and wants to reach point B, 8 km down stream on opposite side. The river is 3 km wide. He can row 6 km/h and run 8 km/h. Where should he land to reach B for least.

\[
\begin{align*}
\text{running dist: } & 8 - x \\
\text{rowing dist: } & \sqrt{9 + x^2} \\
\text{speed} & = \frac{\text{dist}}{\text{time}} = \frac{\text{dist}}{\text{time}} = \text{speed}
\end{align*}
\]

\[\frac{dy}{dt} = \frac{1}{8} (8 - x) + \frac{1}{6} \sqrt{9 + x^2}\]

\[|y'| = 0 \Rightarrow x = \frac{9}{\sqrt{7}}\]

Panel 8

A store is selling 200 DVD burners a week at $350 each. Market research shows that each $10 rebate will sell 20 additional units per week. How large should rebate be to maximize the revenue.

\[x = \# \text{ units sold per week}, \quad p(x) = \text{price per unit}\]

\[R(x) = x \cdot p(x) = \text{revenue}\]

\[p(x) = 350 - \frac{10}{100} (x - 200) = 350 - \frac{1}{10} x\]

\[R(x) = x \cdot (350 - \frac{1}{10} x) = 350x - \frac{1}{10} x^2\]

\[R'(x) = 350 - x = 0 \Rightarrow x = 350\]

\[p(350) = 225\]
Anti-derivatives

A function $F$ is called anti-derivative of $f$ if

$$ F' = f $$

- $f(x) = x^2 \implies F(x) = \frac{1}{3}x^3$ also $g(x) = \frac{1}{3}x^2 + 1$

Any others?

Find $F$ for

- $\sin(x)$
- $\cos(x)$
- $x^{10}$
- $x^{-3}$
- $\sqrt{x}$

$$ f = 12x^2 + 6x - 2 \quad \text{and} \quad f' $$