Panel 1

Next Quiz

Related Rates ✓

Linear Approx \( f(x) \approx f(a) + f'(a)(x-a) \) for \( x \) close to \( a \)

Error Estimates

Panel 2

Page 131, 410

Ship A is 150 km west of Ship B at noon. A sails east at 35 km/h. B sails north at 65 km/h. Distance changing at 4 pm.

\[ d^2 = x^2 + y^2 \]

\[ 2d \frac{dx}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt} \]

\[ \frac{dy}{dt} = 25, \quad y = 100 \]
Panel 3

\[(2.001)^2 \approx 4 \cdot (2.001 - 2) + 4 \cdot (2.001)
\]

\[f''(a) = 2x \quad \Rightarrow \quad x^2 \approx 4 \cdot (x - 2) + 4\]

\[a = 2 \quad \Rightarrow \quad (2.001)^2 \approx 4 \cdot (2.001 - 2) + 4 = 4.001 + 4 = 4.004\]

\[\sqrt{99.9} \approx \sqrt{100 - 0.1}\]

\[f''(a)\]

Panel 4

How many square inches does my screen have?

measure \(l = 10\) with error 0.2 cm

What is error in area?

\[A = l^2, \quad \frac{dA}{dl} = 2l \Rightarrow dA = 2l \cdot dl\]

\[\Rightarrow dA = 2 \cdot l \cdot dl = 2 \cdot 10 \cdot 0.2 = 20.02 = 4\]

\[A = 100\]

relative error in \(A\) in \(\frac{dA}{A} = \frac{4}{100} = 0.04 = 4\%\)

\[\text{in } l \quad \text{in } \frac{dl}{l} = \frac{0.2}{10} = 0.02 = 2\%\]
took a quiz like Quiz 5 again,

and also quiz #4

Panel 6

Last time:

relative max and min

absolute max and min

Extreme Value Theorem: Every continuous function on a closed interval \([a,b]\) has abs. max + min.

Fermat's Theorem: If \(f\) has rel. max or min at \(x=c\), and \(f'(c)\) exists, then \(f'(c) = 0\)

critical points: where \(f' = 0\) or \(f'\) d.n.e.
Panel 7

**How to find local extrema (min or max)**

1.) \( f' = 0 \) or \( f' \) d.n.e. \( \Rightarrow \) critical

2.) Make table using critical points

3.) Decide on signs of \( f' \) to conclude about \( f \).

Example: \( f(x) = 3x^4 - 4x^3 - 12x^2 + 5 \)

1.) \( f'(x) = 12x^3 - 12x^2 - 24x = 0 \)

\[ f' = 12x(x^2 - x - 2) = 0 \]

\[ x = 0, \quad x = 2, \quad x = -1 \]

\[ \uparrow \quad \uparrow \quad \downarrow \]

\[ \text{max} \quad \text{min} \quad \text{min} \]

Panel 8

Find local extrema

\( f(x) = x^{\frac{3}{5}} (4-x) \)

\[ f'(x) = \frac{3}{5} x^{\frac{3}{5}} (4-x) + x^{\frac{3}{5}} (-1) = 0 \]

\[ \frac{3}{5} x^{\frac{3}{5}} (4-x) = x^{\frac{3}{5}} \]

\[ 3(4-x) = 5x \]

\[ 12 - 3x = 5x \]

\[ 12 = 8x \]

\[ x = \frac{3}{2} \text{ in critical. Also } x = 0 \]

\[ \frac{0}{x} \]

\( f' \)

\[ \uparrow \quad \uparrow \quad \downarrow \]

\[ x = 0 \text{ no extrema} \]

\[ x = \frac{3}{2} \text{ is max.} \]
Panel 9

\[ f'(x) = \frac{3}{5} x^2 e^{x} (4-x) + \frac{3}{e^x} (-1) = \]

\[ = \frac{3}{5} (4-x) - \frac{3}{e^x} = \frac{3}{5} \frac{12 - 8x}{e^x} = \frac{12 - 8x}{5 e^x} \]

\[ x = -1 : \quad \frac{t}{t} \]

\[ x = 1 : \quad t \]

\[ x = \pm 10 \text{ and } (x) \quad \]