d. Find the output required to obtain a profit of $10,000.

**Solution:** In order to obtain a profit of $10,000, we have

\[ \text{profit} = \text{total revenue} - \text{total cost} \]

\[ 10,000 = 8q - \left( \frac{22}{9} q + 5000 \right) \]

\[ 15,000 = \frac{50}{9} q \]

\[ q = 2700 \]

Thus, 2700 units must be produced.

**EXAMPLE 4**  
**Break-Even Quantity**

Determine the break-even quantity of XYZ Manufacturing Co., given the following data: total fixed cost, $1200; variable cost per unit, $2; total revenue for selling q units, \( y_{TR} = 100\sqrt{q} \).

**Solution:** For q units of output,

\[ y_{TR} = 100\sqrt{q} \]

\[ y_{TC} = 2q + 1200 \]

Equating total revenue to total cost gives

\[ 100\sqrt{q} = 2q + 1200 \]

\[ 50\sqrt{q} = q + 600 \]

Squaring both sides, we have

\[ 2500q = q^2 + 1200q + (600)^2 \]

\[ 0 = q^2 - 1300q + 360,000 \]

By the quadratic formula,

\[ q = \frac{1300 \pm \sqrt{1690,000}}{2} \]

\[ q = \frac{1300 \pm 500}{2} \]

\[ q = 400 \quad \text{or} \quad q = 900 \]

Although both \( q = 400 \) and \( q = 900 \) are break-even quantities, observe in Figure 3.49 that when \( q > 900 \), total cost is greater than total revenue, so there will always be a loss. This occurs because here total revenue is not linearly related to output. Thus, producing more than the break-even quantity does not necessarily guarantee a profit.

**Now Work Problem 9**

**PROBLEMS 3.6**  
In Problems 1–8, you are given a supply equation and a demand equation for a product. If \( p \) represents price per unit in dollars and \( q \) represents the number of units per unit of time, find the equilibrium point. In Problems 1 and 2, sketch the system.

1. Supply: \( p = \frac{2q}{60} + 3 \), Demand: \( p = -\frac{3}{10} q + 11 \)
2. Supply: \( p = \frac{100}{q} + 4 \), Demand: \( p = -\frac{3}{200} q + 9 \)
3. Supply: \( 35q - 2p + 250 = 0 \), Demand: \( 65q + p - 537.5 = 0 \)
4. Supply: \( 246p - 3.25q - 2460 = 0 \), Demand: \( 410p + 3q - 14,452.5 = 0 \)
5. Supply: \( p = 2q + 20 \), Demand: \( p = 200 - 2q^2 \)
6. Supply: \( p = (q + 12)^2 \), Demand: \( p = 644 - 6q - q^2 \)
7. Supply: \( p = \sqrt{q} + 10 \), Demand: \( p = 20 - q \)
8. Supply: \( p = \frac{1}{4} q + 6 \), Demand: \( p = \frac{2240}{q + 12} \)
Section 3.6 Applications of Systems of Equations

21. Business (a) Find the break-even points for company X, which sells all it produces, if the variable cost per unit is $3, fixed costs are $2, and $y_T = 5\sqrt{q}$, where $q$ is the number of thousands of units of output produced.
(b) Graph the total revenue curve and the total cost curve in the same plane.
(c) Use your answer in (a) to report the quantity interval in which maximum profit occurs.

22. Business A company has determined that the demand equation for its product is $p = 1000/q$, where $p$ is the price per unit for $q$ units produced and sold in some period. Determine the quantity demanded when the price per unit is (a) $4$, (b) $2$, and (c) $0.50$. For each of these prices, determine the total revenue that the company will receive. What will be the revenue regardless of the price? [Hint: Find the revenue when the price is $p$ dollars.]

23. Business Using the data in Example 1, determine how the original equilibrium price will be affected if the company is given a government subsidy of $1.50 per unit.

24. Business The Monroe Forging Company sells a corrugated steel product to the Standard Manufacturing Company and is in competition on such sales with other suppliers of the Standard Manufacturing Co. The vice president of sales of Monroe Forging Co. believes that by reducing the price of the product, a 40% increase in the volume of units sold to the Standard Manufacturing Co. could be secured. As the manager of the cost and analysis department, you have been asked to analyze the proposal of the vice president and submit your recommendations as to whether it is financially beneficial to the Monroe Forging Co. You are specifically requested to determine the following:
(a) Net profit or loss based on the pricing proposal
(b) Unit sales volume under the proposed price that is required to make the same $40,000 profit that is now earned at the current price and unit sales volume

Use the following data in your analysis:

<table>
<thead>
<tr>
<th>Unit price</th>
<th>Current Operations</th>
<th>Proposal of Vice President of Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.50</td>
<td>$2.00</td>
<td></td>
</tr>
<tr>
<td>Unit sales volume</td>
<td>200,000 units</td>
<td>280,000 units</td>
</tr>
<tr>
<td>Variable cost</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$350,000</td>
<td>$400,000</td>
<td></td>
</tr>
<tr>
<td>Per unit</td>
<td>$1.75</td>
<td>$1.75</td>
</tr>
<tr>
<td>Fixed cost</td>
<td>$110,000</td>
<td>$110,000</td>
</tr>
<tr>
<td>Profit</td>
<td>$40,000</td>
<td>$7</td>
</tr>
</tbody>
</table>

25. Business Suppose products A and B have demand and supply equations that are related to each other. If $q_A$ and $q_B$ are the quantities produced and sold of A and B, respectively, and $p_A$ and $p_B$ are their respective prices, the demand equations are

\[
q_A = 7 - p_A + p_B
\]

and

\[
q_B = 24 + p_A - p_B
\]

and the supply equations are

\[
q_A = -3 + 4p_A - 2p_B
\]

and

\[
q_B = -5 - 2p_A + 4p_B
\]

Eliminate $q_A$ and $q_B$ to get the equilibrium prices.