CHAPTER 3  Lines, Parabolas, and Systems

Figure 3.12 Lines parallel and perpendicular to \( y = 3x + 1 \) (Example 9).

The slope of a line perpendicular to \( y = 3x + 1 \) must be \(-\frac{1}{3}\) (the negative reciprocal of 3). Using a point-slope form, we get:

\[
y - (-2) = -\frac{1}{3}(x - 3)
\]
\[
y + 2 = -\frac{1}{3}x + 1
\]
\[
y = -\frac{1}{3}x - 1
\]

PROBLEMS 3.1

In Problems 1–8, find the slope of the straight line that passes through the given points.

1. \((3, 2), (7, 10)\)
2. \((-2, 10), (5, 3)\)
3. \((6, -2), (8, -3)\)
4. \((2, -4), (3, -4)\)
5. \((5, 3), (5, -8)\)
6. \((0, -40), (3, 6)\)
7. \((5, -2), (4, -2)\)
8. \((1, -7), (9, 0)\)

In Problems 9–24, find a general linear equation \((Ax + By + C = 0)\) of the straight line that has the indicated properties, and sketch each line.

9. Passes through \((-1, 7)\) and has slope \(-5\)
10. Passes through the origin and has slope 75
11. Passes through \((-5, 5)\) and has slope \(-\frac{1}{2}\)
12. Passes through \((-\frac{1}{3}, 5)\) and has slope \(\frac{3}{5}\)
13. Passes through \((-6, 1)\) and \((1, 4)\)
14. Passes through \((5, 2)\) and \((6, -4)\)
15. Passes through \((-3, -4)\) and \((-2, -8)\)
16. Passes through \((0, 0)\) and \((-3, -2)\)
17. Has slope 2 and \(y\)-intercept \(-4\)
18. Has slope 5 and \(y\)-intercept \(-7\)
19. Has slope \(-1\) and \(y\)-intercept \(3\)
20. Has slope 0 and \(y\)-intercept \(-\frac{1}{2}\)
21. Is horizontal and passes through \((-2, -5)\)
22. Is vertical and passes through \((-1, -1)\)
23. Passes through \((2, -3)\) and is vertical
24. Passes through the origin and is horizontal

In Problems 25–34, find, if possible, the slope and \(y\)-intercept of the straight line determined by the equation, and sketch the graph.

25. \(y = 4x - 6\)
26. \(x + 9 = 2\)
27. \(3x + 5y - 9 = 0\)
28. \(y + 4 = 7\)
29. \(x = -5\)
30. \(x = -5y + 3\)
31. \(y = -2x\)
32. \(y - 7 = 3(x - 4)\)
33. \(y = 3\)
34. \(6y - 24 = 0\)

In Problems 35–40, find a general linear form and the slope-intercept form of the given equation.

35. \(2x = 5 - 3y\)
36. \(5x - 2y = 10\)
37. \(4x + 9y - 5 = 0\)
38. \(3(x - 4) - 7(y + 1) = 2\)
39. \(\frac{x}{2} + \frac{2y}{3} = -\frac{3}{4}\)
40. \(y = \frac{1}{300}x + 8\)

In Problems 41–50, determine whether the lines are parallel, perpendicular, or neither.

41. \(y = -5x + 7, y = -5x - 3\)
42. \(y = 4x + 3, y = 5 + 4x\)
43. \(y = 5x + 2, -5x + y = -3\)
44. \(y = x, y = -x\)
45. \(x + 3y + 5 = 0, y = -3x\)
46. \(x + 2y = 0, x + 4y - 4 = 0\)
47. \(y = 3, x = -\frac{1}{3}\)
48. \(x = 3, x = -3\)
49. \(3x + y = 4, x + 3y + 1 = 0\)
50. \(x - 2 = 3, y = 2\)
In Problems 51–60, find an equation of the line satisfying the given conditions. Give the answer in slope-intercept form if possible.

51. Passing through (2, 3) and parallel to \( y = 4x + 3 \)
52. Passing through \((-2, -8)\) and parallel to \( x = -4 \)
53. Passing through \((2, 1)\) and parallel to \( y = 2 \)
54. Passing through \((3, -4)\) and parallel to \( y = 3 + 2x \)
55. Perpendicular to \( y = 3x - 5 \) and passing through \((3, 4)\)
56. Perpendicular to \(3x + 2y = 4\) and passing through \((3, 1)\)
57. Passing through \((5, 2)\) and perpendicular to \( y = -3 \)
58. Passing through \((-4, -5)\) and perpendicular to the line \(3y = 2x + 3\)
59. Passing through \((-7, -5)\) and parallel to the line \(2x + 3y + 6 = 0\)
60. Passing through \((-4, 10)\) and parallel to the \(y\)-axis

61. A straight line passes through \((-1, -2)\) and \((4, 1)\). Find the point on it that has an \(x\)-coordinate of 3.

62. A straight line has slope 3 and \(y\)-intercept \((-2, 0)\). Does the point \((-1, -2)\) lie on the line?

63. Stock. In 1996, the stock in a computer hardware company traded for \$37 per share. However, the company was in trouble and the stock price dropped steadily, to \$8 per share in 2006. Draw a line showing the relationship between the price per share and the year in which it traded for the time interval [1996, 2006], with years on the \(x\)-axis and price on the \(y\)-axis. Find and interpret the slope.

In Problems 64–65, find an equation of the line describing the following information.

64. Home Runs. In one season, a major league baseball player has hit 14 home runs by the end of the third month and 20 home runs by the end of the fifth month.

65. Business. A delicatessen owner starts her business with debts of \$100,000. After operating for five years, she has accumulated a profit of \$40,000.

66. Due Date. The length, \(L\), of a human fetus more than 12 weeks old can be estimated by the formula \(L = 1.53t - 6.7\), where \(L\) is in centimeters and \(t\) is in weeks from conception. An obstetrician uses the length of a fetus, measured by ultrasound, to determine the approximate age of the fetus and establish a due date for the mother. The formula must be rewritten to result in an age, \(t\), given a length, \(L\). Find the slope and \(L\)-intercept of the equation.

67. Discuss Throw. A mathematical model can approximate the winning distance for the Olympic discus throw by the formula \(d = 191.4 + t\), where \(d\) is in feet and \(t\) = 0 corresponds to the year 1948. Find a general linear form of this equation.

68. Campus Map. A coordinate map of a college campus gives the coordinates \((x, y)\) of three major buildings as follows: computer center, \((3.5, -1.5)\); engineering labs, \((0.5, 0.5)\); and library \((-1.2, -2.5)\). Find the equations (in slope-intercept form) of the straight-line paths connecting (a) the engineering lab with the computer center and (b) the engineering lab with the library. Are these two paths perpendicular to each other?

69. Geometry. Show that the points \(A(0, 0)\), \(B(0.4, 0.7)\), \(C(2, 3)\), and \(D(2, 7)\) are the vertices of a parallelogram. (Opposite sides of a parallelogram are parallel.)

70. Approach Angle. A small plane is landing at an airport with an approach angle of 45 degrees, or slope of \(-1\). The plane begins its descent when it has an elevation of 3600 feet. Find the equation that describes the relationship between the craft's altitude and distance traveled, assuming that at distance 0 it starts the approach angle. Graph your equation on a graphing calculator. What does the graph tell you about the approach if the airport is 3800 feet from where the plane starts its landing?

71. Cost Equation. The average daily cost, \(C\), for a room at a city hospital has risen by \$50.82 per year for the years 1990 through 2000. If the average cost in 1990 was \$1128.50, what is an equation which describes the average daily cost during this decade, as a function of the number of years, \(T\), since 1990?

72. Revenue Equation. A small business predicts its revenue growth by a straight-line method with a slope of \$50,000 per year. In its fifth year, it had revenues of \$330,000. Find an equation that describes the relationship between revenues, \(R\), and the number of years, \(T\), since it opened for business.

73. Graph \(y = -0.9x + 7.3\) and verify that the \(y\)-intercept is 7.3.

74. Graph the lines whose equations are
   \[ y = 1.5x + 1 \]
   \[ y = 1.5x - 1 \]

    and

    \[ y = 1.5x + 2.5 \]

What do you observe about the orientation of these lines? Why would you expect this result from the equations of the lines themselves?

75. Graph the line \(y = 7.1x + 5.4\). Find the coordinates of any two points on the line, and use them to estimate the slope. What is the actual slope of the line?

76. Show that if a line has \(x\)-intercept \(a\) and \(y\)-intercept \(b\), both different from 0, then \(\frac{a}{b} + \frac{b}{a} - 1 = \text{an equation of the line.}

3.2 Applications and Linear Functions

Many situations in economics can be described by using straight lines, as evidenced by Example 1.

**EXAMPLE 1 Production Levels**

Suppose that a manufacturer uses 100 lb of material to produce products A and B, which require 4 lb and 2 lb of material per unit, respectively. If \(x\) and \(y\) denote the number of units produced of A and B, respectively, then all levels of production are given by the