Panel 1

Had fun with Derivatives ....

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Panel 2

Anti-Derivatives

Know: start with f, find f'

Opposite: given a function f, find F such that $F' = f$

F is called anti-derivative of f.

Ex: $f(x) = 2x$. Then $F(x) = x^2$ YES, because $\frac{d}{dx}(x^2) = 2x$

$F' = f$

Notation: The anti derivative of a function $f(x)$ is

$$F(x) = \int f(x)\,dx$$

"integral of $f(x)\,dx$"

Ex: $\int 2x\,dx = x^2$

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Panel 3

Ex:

$$\int 1\,dx = x$$

$$\int x\,dx = \frac{1}{2}x^2 \qquad \frac{d}{dx}\left(\frac{1}{2}x^2\right) = \frac{1}{2}\cdot 2x = x$$

$$\int x^5\,dx = \frac{1}{6}x^6 \qquad \frac{d}{dx}\left(\frac{1}{6}x^6\right) = \frac{1}{6}\cdot 6x^5$$

$$\int x^8\,dx = \frac{1}{9}x^9$$

$$\int \sqrt{x}\,dx = \int x^{1/2}\,dx = \frac{2}{3}x^{3/2}, \quad \frac{d}{dx}\frac{2}{3}x^{3/2} = \cancel{\frac{2}{3}}\cdot\cancel{\frac{3}{2}}x^{1/2}$$

$$\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx = -\frac{1}{2}x^{-2}$$

$$\int \frac{1}{\sqrt[3]{x^4}}\,dx = \int x^{-4/3}\,dx = -3x^{-1/3}$$

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Panel 4

Anti Power Rule:

$$\int x^p\,dx = \frac{1}{p+1}x^{p+1} + C$$

Ex: $\int x^6\,dx = \frac{1}{7}x^7 + C$

Rule: Anti derivatives usually include an arbitrary constant.
because $d/dx\,(\text{const}) = 0$

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Panel 5

More:

$\int x^2 + 2x \, dx = \frac{1}{3}x^3 + x^2 + C$

$\frac{1}{3}x^3 + 2\frac{1}{2}x^2 + c$

$\int 2\sqrt[5]{x^4} - 7x^3 + 10 \, dx$

$\int 2 x^{4/5} - 7x^3 + 10x^0 \, dx =$

$2\frac{5}{9}x^{9/5} - 7\frac{1}{4}x^4 + 10x + C$

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Panel 6

Even more:

① Find a function y such that $y' = 8x - 4$ and $y(2) = 5$

initial condition

Want. $y = \int 8x - 4 \, dx = 8\frac{1}{2}x^2 - 4x + C$

$= \boxed{4x^2 - 4x + C}$

Also: $y(2) = 5$

$y(2) = 4 \cdot (2)^2 - 4(2) + C = 8 + C = 5 \Rightarrow C = -3$

$\Rightarrow y(x) = 4x^2 - 4x - 3$

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Panel 7

① Find a function y such that $y'' = x^2 - 6$ and also $y'(0) = 2$ and $y(1) = -1$

Want: $y' = \int x^2 - 6 \, dx = \frac{1}{3}x^3 - 6x + C$

$y'(0) = \frac{1}{3}0^3 - 6\cdot 0 + C = 2 \Rightarrow C = 2$

$y'(x) = \frac{1}{3}x^3 - 6x + 2$

$y(x) = \int \frac{1}{3}x^3 - 6x + 2 \, dx = \frac{1}{3}\frac{1}{4}x^4 - 6\frac{1}{2}x^2 + 2x + C$

$= \frac{1}{12}x^4 - 3x^2 + 2x + C$

$y(1) = \frac{1}{12} - 3 + 2 + C = -1$

$-\frac{11}{12} + C = -1 \Rightarrow C = -\frac{1}{12}$

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Panel 8

Ex: Suppose the marginal revenue function for a product is $\frac{dr}{dq} = 2000 - 20q - 3q^2$

Find the demand function

Easy: $R(q) = \int 2000 - 20q - 3q^2 \, dq =$

$= 2000q - 20\frac{1}{2}q^2 - 3\frac{1}{3}q^3 + C$

$= 2000q - 10q^2 - q^3 + \cancel{C}$, $R(0) = \$0 = C$

$\boxed{R(q) = p(q)\cdot q} = 2000q - 10q^2 - q^3 \Rightarrow p(q) = 2000 - 10q - q^2$

demand $= (2000 - 10q - q^2)q$

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Panel 9

Rules of Integration

$$\int x^p\,dx = \frac{1}{p+1}x^{p+1} + C \quad , p \neq -1$$

new

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C$$

✓ sums / differences : do seperately

✓ $\#\cdot$ function = keep #, find antideriv. of function

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Panel 10

e) $\int \frac{2x^2}{7} - \frac{8}{3}x^4\,dx = \frac{2}{7}\cdot\frac{1}{3}x^3 - \frac{8}{3}\frac{1}{5}x^5 + C$

f) $\int (x^2+5)(x-3)\,dx = \int x^3 - 3x^2 + 5x - 15\,dx = \frac{1}{4}x^4 - 3\frac{1}{3}x^3 + 5\frac{1}{2}x^2 - 15x + C$

g) $\int \frac{2}{\sqrt{x^3}} - \frac{\sqrt[4]{x^5}}{8}\,dx = \int 2x^{-3/2} - \frac{1}{8}x^{5/4}\,dx =$

$= 2\cdot(-2)x^{-1/2} - \frac{1}{8}\frac{4}{9}x^{9/4} + C$

h) $\int (3x+2)^3\,dx$

Foil first

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Panel 11

a) $\int \frac{7}{x}\,dx = 7\ln|x| + C$

b) $\int 8e^x\,dx = 8e^x + C$

c) $\int 5e^x + \frac{1}{3x}\,dx = 5e^x + \frac{1}{3}\ln|x| + C$

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Panel 12

If the fixed costs for producing Acme Quality Widgets is \$2000 and the marginal cost is $c'(q) = 0.08q^2 - 1.6q + 6.5$, find the cost for producing 25 units.

$$C(q) = \int 0.08q^2 - 1.6q + 6.5\,dq =$$

$$= 0.08\frac{1}{3}q^3 - 1.6\frac{1}{2}q^2 + 6.5q + C$$

$$C(0) = 2000 = C \Rightarrow C = 2000$$

$$C(q) = \frac{0.08}{3}q^3 - 0.8q^2 + 6.5q + 2000$$

Answer: $C(25)$

calculator

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Panel 13

Next time: Definite Integral

$\int f(x)\,dx$ is anti-derivative, or indefinite integral.

What: $\int_a^b f(x)\,dx$ = integral of $f(x)$ from $x=a$ to $x=b$

= definite integral.

Ex: $\int_0^1 x^2\,dx$ = Next time

$\boxed{\text{Quiz on Wed}}$

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