Panel 1

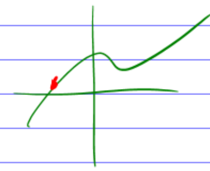
Different Terms for "Derivative":

Definition $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

Rules: Power Rule, $\frac{d}{dx} e^x = e^x$ $\quad \frac{d}{dx} \ln(x) = \frac{1}{x}$

Rate of change

Marginal Revenue, Cost, Profit



Slope of tangent

Physics: velocity

decreasing, increasing

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Panel 2

Ex: $C(x) = 5 \cdot \ln(x) + 7e^x - 5x^4 + 2\overset{x^{1/2}}{\sqrt{x}} + \underbrace{\ln(\pi) + \sqrt{2} + e^7 + 9}$

Find the rate of change of the marginal cost function.

1. Marginal cost. $C'(x) = 5 \cdot \overset{x^{-1}}{\frac{1}{x}} + 7e^x - 20x^3 + 2 \cdot \frac{1}{2} x^{-1/2}$ $\boxed{\text{say } C'(10) > 0}$

Rate of change of $C'(x) = C''(x) = -5x^{-2} + 7e^x - 60x^2 - \frac{1}{2} x^{-3/2}$

Recall: $\frac{d}{dx} 5x^3 = 15x^2$

$\frac{d}{dx} 7\sqrt{x} = \frac{d}{dx} 7x^{1/2} = 7 \cdot \frac{1}{2} x^{-1/2}$

$\frac{d}{dx} \frac{9}{x^3} = \frac{d}{dx} 9x^{-3} = 9 \cdot (-3) x^{-4}$

$\frac{d}{dx} 5 \sqrt[3]{x^2} = 5 \left(x^2\right)^{1/3}$

$\frac{d}{dx} \left(5x^{2/3}\right)$

$5 \cdot \frac{2}{3} x^{-1/3}$

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Panel 3

Ex: If $c(q) = 0.2q + 2 + \frac{500}{q}$ is the average cost then find. average cost $c(q) = \frac{C(q)}{q}$

a) $C(q) = q \cdot c(q) = 0.2q^2 + 2q + 500$

b) Fixed cost: $C(0) = 500$

c) Marginal cost: $C'(q) = 0.4q + 2$

d) Rate of change of marginal cost

$C''(q) = 0.4$

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Panel 4

Name: ______

Quiz #7

① If $P(x) = 10\ln(x) + e^x$ is a profit function

a) find the marginal profit for $x = 1$

$P'(x) = 10 \cdot \frac{1}{x} + e^x \quad \Rightarrow P'(1) = 10 \cdot \frac{1}{1} + e^1 = 10 + e > 0$

b) Should you increase or decrease production from its current level of $x = 1$?

Profit increases at $x=1$ $\Rightarrow$ increase production

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Panel 5

(2) Find the indicated derivative for the function:

a) $f(x) = 3x^2 - 2\sqrt{x} + 3 \cdot \ln(x)$; find $f''(x)$ ($\sqrt{x} = x^{1/2}$)

$f'(x) = 6x - 2 \cdot \frac{1}{2} x^{-1/2} + 3 \cdot \frac{1}{x} = 6x - x^{-1/2} + 3x^{-1}$

$f''(x) = 6 - (-\frac{1}{2}) x^{-3/2} + 3(-1) x^{-2}$

b) $f(x) = 4x^3 - 3x^2 + 7x - 9$; find $f^{(10)}(x)$, the 10^{th}-deriv

$f'(x) = 12x^2 - 6x + 7$

$f''(x) = 24x - 6$

$f'''(x) = 24$ $\Rightarrow f^{(10)}(x) = 0$

$f^{(4)}(x) = 0$

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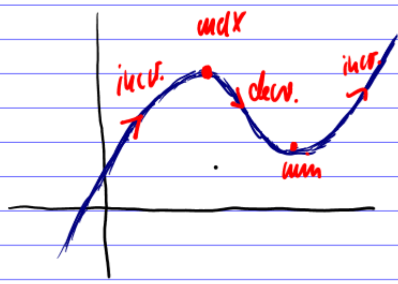
Panel 6

Move appl. of derivative: Max / Min of a Function



f' is positive.

$\Rightarrow$ f is increasing

f' is negative

$\Rightarrow$ f is decreasing

Theorem: If f has max or min at $x = c$,
then $f'(c) = 0$ or $f'(c)$ is undefined.

Def: critical points are where $f'(x) = 0$ or $f'(x)$ is undefined.

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Panel 7

How to Find local Max / Min

either max or min

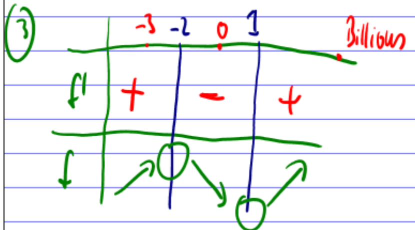
Ex: $f(x) = 2x^3 + 3x^2 - 12x - 3$ Find all local extrema

① $f'(x) = 6x^2 + 6x - 12$

② $0 = 6x^2 + 6x - 12$

critical

$= 6(x^2 + x - 2) = 6(x+2)(x-1)$, $\underline{x = -2, 1}$

③ 

$x = -2$ max

$x = 1$ min

Recipe

① $f'(x)$

② $f'(x) = 0$ or undefined (critical points)

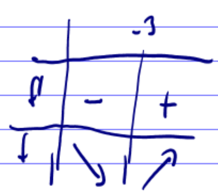
③ Make a table of signs of f' between all critical points.

④ Read off the answers

Panel 8

Ex: $f(x) = x^2 + 6x - 8$ Find local max / min

$f'(x) = 2x + 6 = 0 \Rightarrow x = -3$ critical point.



$x = -3$ is a min.

$f(x) = ax^2 + bx + c$

$f'(x) = 2ax + b = 0$

$\boxed{x = -b/2a}$

Panel 9

Ex: Suppose $C(x) = \frac{360000}{x} + 4x$ is a cost function based on the inventory $x > 0$. How much inventory should you carry to minimize the cost?

$C'(x) = -360000x^{-2} + 4 = -\frac{360000}{x^2} + 4 = 0$

$4 = \frac{360000}{x^2} \iff x^2 = \frac{360000}{4} = 90000$

$x = \pm 300$

		+300	Billion
C'	$-$		$+$
C	$\searrow$		$\nearrow$

$x = 300$ gives min. cost, and that min cost is

$C(300) = \frac{360000}{300} + 4 \cdot 300$

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