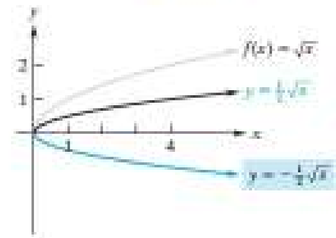

FIGURE 2.37 To graph $y = -\frac{1}{2}\sqrt{x}$, shrink $y = \sqrt{x}$ and reflect result about x-axis.

the x-axis by a factor of $\frac{1}{2}$ (transformation 8, Table 2.2; see Figure 2.37). Second, the minus sign in $y = -\frac{1}{2}\sqrt{x}$ causes a reflection in the graph of $y = \frac{1}{2}\sqrt{x}$ about the x-axis (transformation 5, Table 2.2; see Figure 2.37).

Now Work Problem 5 ◁

PROBLEMS 2.7

In Problems 1–12, use the graphs of the functions in Figure 2.34 and transformation techniques to plot the given functions.

1. $y = x^3 - 1$ **2.** $y = -x^2$ **3.** $y = \frac{1}{x-2}$

4. $y = -\sqrt{x-2}$ **5.** $y = \frac{2}{3x}$ **6.** $y = |x| - 2$

7. $y = |x+1| - 2$ **8.** $y = -\frac{1}{3}\sqrt{x}$ **9.** $y = 2 + (x+3)^3$

10. $y = (x-1)^2 + 1$ **11.** $y = \sqrt{-x}$ **12.** $y = \frac{5}{2-x}$

In Problems 13–16, describe what must be done to the graph of $y = f(x)$ to obtain the graph of the given equation.

13. $y = -2f(x+3) + 2$ **14.** $y = 2(f(x-1) - 4)$

15. $y = f(-x) - 5$ **16.** $y = f(3x)$

17. Graph the function $y = \sqrt[3]{x} + k$ for $k = 0, 1, 2, 3, -1, -2$, and -3. Observe the vertical translations compared to the first graph.

18. Graph the function $y = \sqrt[5]{x+k}$ for $k = 0, 1, 2, 3, -1, -2$, and -3. Observe the horizontal translations compared to the first graph.

19. Graph the function $y = kx^3$ for $k = 1, 2, \frac{1}{2}$, and 3. Observe the vertical stretching and shrinking compared to the first graph. Graph the function for $k = -2$. Observe that the graph is the same as that obtained by stretching the reflection of $y = x^3$ about the x-axis by a factor of 2.

Objective

To discuss functions of several variables and to compute function values. To discuss three-dimensional coordinates and sketch simple surfaces.

2.8 Functions of Several Variables

When we defined a *function* $f : X \longrightarrow Y$ from X to Y in Section 2.1, we did so for *sets* X and Y without requiring that they be sets of numbers. We have not often used that generality yet. Most of our examples have been functions from $(-\infty, \infty)$ to $(-\infty, \infty)$. We also saw in Section 2.1 that, for sets X and Y, we can construct the new set $X \times Y$ whose elements are ordered pairs (x, y) with x in X and y in Y. It follows that, for any three sets X, Y, and Z, the notion of a function $f : X \times Y \longrightarrow Z$ is already covered by the basic definition. Such an f is simply a rule which assigns to each element (x, y) in $X \times Y$ at most one element of Z, denoted by $f((x, y))$. There is general agreement that in this situation one should drop a layer of parentheses and write simply $f(x, y)$ for $f((x, y))$. Do note here that even if each of X and Y are sets of numbers, say $X = (-\infty, \infty) = Y$, *is not a number.*

The *graph* of a function $f : X \longrightarrow Y$ is the subset of $X \times Y$ consisting of all ordered pairs of the form $(x, f(x))$, where x is in the domain of f. It follows that the graph of a function $f : X \times Y \longrightarrow Z$ is the subset of $(X \times Y) \times Z$ consisting of all ordered pairs of the form $((x, y), f(x, y))$, where (x, y) is in the domain of f. The ordered pair $((x, y), f(x, y))$ has its first coordinate given by (x, y), itself an ordered pair, while its