

MATH 1203 – Practice Exam 3

This is a practice exam only. The actual exam may differ from this practice exam.

Please provide brief answers to the following questions:

- a) If you are using a t-distribution with $df = 11$ for a statistical test at the $\alpha = 0.05$ level, then the corresponding number t_α will be what?

$t_\alpha = 2.201$

- b) If you are using a t-distribution with $df = 11$ for a statistical test at the $\alpha = 0.05$ level, then the corresponding number t_α will be what?

Same question, same answer

- c) If you are using a t-distribution with $df = 11$ for a statistical test, and the number t_0 you compute is $t_0 = 2.10$, whereas the number t_1 you look up is $t_1 = 2.41$. What is your conclusion for the corresponding test?

If $t_0 > t_\alpha$, then reject H_0 . Since $2.10 > 2.41$ is not true, the test is inconclusive

- d) If you are using z-distribution for a statistical test at the usual 0.05 ($= 5\%$) level of significance, the number z_0 you compute is $z_0 = 2.89$, and the corresponding p-value for that value of z_0 is 0.038 . What is your conclusion for the corresponding test?

If $p < 0.05$ then reject H_0 . Since $p = 0.038$, you indeed reject H_0

- e) Someone is interested in designing a statistical test for the mean of a population. In deciding whether to use a test based on the t-distribution or a test based on the standard normal distribution, what is the deciding factor?

The sample size

You are conducting a 1-tailed statistical test for the population mean at the $\alpha = 0.05$ level. The null hypothesis is $H_0 = 17.1$, while the alternative hypothesis is $H_a > 17.1$. The sample size is large enough to use a normal distribution, and the statistics for the sample turns out to be $z_0 = 2.045$. What is your conclusion?

$p = 2 * P(z_0 > 2.045) = 2 * 0.0207 = 0.0414$ is less than 0.05 , so you reject the null hypothesis H_0

A statistical test for the population mean at the $\alpha = 0.05$ level results in your rejection of the null hypothesis. Can the null hypothesis still be true? If so, what is the probability that the null hypothesis is true, even though you rejected it?

Yes it can, with probability less than $\alpha = 0.05$. If we computed a p-value, then that number would be the probability that H_0 could be true even though I would reject it.

You were asked to compute a 95% confidence interval. The resulting interval, however, turned out to be too large to be of use to your client. What could you do to achieve a smaller confidence interval?

You could compute a 90% confidence interval, or repeat the experiment with an increased sample size.

On average, do males outperform females in mathematics? To answer this question, psychologists at the University of Minnesota compared the scores of male and female eighth-grade students who took a basic skill math test. A summary of the test scores is displayed below.

	Males	Females
Sample Size	1764	1739
Mean	48.9	48.4
Standard Deviation	12.96	11.85

H₀: mean₁ = mean₂

H₁: mean₁ not equal to mean₂

$z_0 = ((48.9 - 48.4) - 0) / s$, where $s = 0.41948220487302895$, so that

$z_0 = 1.192$

Therefore $p = 2 * P(z > 1.192) = 2 * 0.1170$ is not less than 0.05, so Test is Inconclusive

The Cleveland Casting plant produces iron automotive castings for Ford. When the process is stable, the target pouring temperature of the molten iron is 2,550 degrees. The pouring temperatures for a random sample of 10 crankshafts produced at the plant are listed below. Does the mean pouring temperature differ from the target setting?

2543, 2541, 2544, 2620, 2560, 2559, 2562, 2553, 2552, 2553

We can compute the sample mean $\bar{x} = 2558.7$ and the sample standard deviation $s = 22.7$. Thus:

H₀: mean = 2550

H₁: mean not equal to 2550

$t_0 = (2558.7 - 2550) / (22.7 / \sqrt{10}) = 1.212$

From the t-table we find $t_a = 2.262$

Since $t_0 < t_a$, our test is inconclusive, i.e. there is not enough evidence to suggest anything wrong.

According to USA Today (Dec. 1999) the average age of MSNBC TV News viewers is 50 years. A company wants to market a product for this age group, but wants to ensure that the USA Today study is correct before investing advertisement money. They select 50 US households at random that view MSNBC TV News and find their average age to be 51.3 years with a standard deviation of 7.1 years. Should the company invest in advertising?

H₀: mean = 50

H_a: mean not equal to 50

$z_0 = (51.3 - 50) / (7.1 / \sqrt{50}) = 1.29$

p is certainly not less than 0.05, so this test is inconclusive as well.

The “fear of negative evaluation” (FNE) scores for 11 bulimic female students and 14 normal female students are shown below (the higher the score, the greater the fear of negative evaluation). What is the average FNE score of bulimic female students and that of normal female students? Is there a significant difference between the mean FNE scores?

Bulimic students: 21, 13, 10, 20, 25, 19, 16, 21, 24, 13, 14

Normal students: 13, 6, 16, 13, 8, 19, 23, 18, 11, 19, 7, 10, 15, 20

We compute $\bar{x}_1 = 17.8$, $s_1 = 4.9$ and $\bar{x}_2 = 14.1$, $s_2 = 5.3$

H₀: mean₁ = mean₂

H_a: mean₁ not equal to mean₂

$t_0 = (17.8 - 14.1) / s$ where $s = 2.047$ so that

$t_0 = 1.807$

We look up t_a with $df = (n_1 - 1) + (n_2 - 1) = 10 + 13 = 23$ in the $t_{0.025}$ column to find $t_a = 2.069$

Thus, since our computed number is not bigger than the number we look up the test is inconclusive.

Suppose you want to compare a new method of teaching reading to “slow learners” to the current standard method. You select a random sample of 22 slow learners; 10 of them are taught by the new method and 12 are taught by the standard method, for the same period of time. The reading scores for the two groups were as follows:

New Method	Standard Method
80, 80, 79, 81, 76, 66, 71, 76, 70, 85	79, 62, 70, 68, 73, 76, 86, 73, 72, 68, 75, 66

- What is the difference in average reading scores between the two methods?
- Conduct a test to determine whether the new method is better than the standard method.

We compute $\bar{x}_1 = 76.4$ and $s_1 = 5.8$ as well as $\bar{x}_2 = 72.3$ and $s_2 = 6.65$. Because we will need it soon, let's also compute the combined $s = \sqrt{(s_1^2/n_1 + s_2^2/n_2)} = 2.65$

Question a is $76.4 - 72.3 = 4.1$

H₀: tests are the same (means are equal)

H₁: tests are different (means not equal)

$t_0 = 4.1 / 2.65 = 1.54$

This will again result in an inconclusive test (too bad, sorry)

The lifetimes (in years) of ten automobile batteries of a particular brand are:

2.4	1.9	2.0	2.1	1.8
2.3	2.1	2.3	1.7	2.0

Estimate the mean lifetime for all batteries, using a 95% confidence interval.

Confidence intervals have low priority, so I'll skip this for now. Maybe I'll add the answer later.

A large supermarket chain sells longhorn cheese in one-pound (= 16 ounces) packages. As a city inspector you weigh 81 randomly selected packages of cheese and note that the sample mean is 15.58 ounces, with a standard deviation of 1.44 ounces. You therefore suspect that the chain is miss-labeling the cheese and that the actual weight of a package is less than 16 ounces. Use this data to test your suspicion against the null hypothesis that the average weight of a package is 16 ounces. Use $\alpha = 0.05$.

H₀: mean = 16

H_a: mean not equal to 16

$z_0 = (15.58 - 16) / (1.44 / \sqrt{81}) = -2.625$

Now we compute $p = 2 * P(z > 2.625) = 2 * 0.0044 = 0.0088 < 0.05$ so we reject the null hypothesis. In other words, we think that the cheeses are mislabeled!

A test was conducted to determine the length of time required for a student to read a specified amount of material. All students were instructed to read at the maximum speed at which they could still comprehend the material. Sixteen students took the test, with the following results (in minutes):

25, 18, 27, 29, 20, 19, 25, 24, 32, 21, 24, 19, 23, 28, 31, 22

Estimate the mean length of time required for all students to read the material, using a 95% confidence interval.

We compute $\bar{x} = 24.18$ and $s = 4.32$. The number k we look up from the t-table for $df = 16 - 1 = 15$ is $k = 2.131$. Thus the 95% confidence interval goes from $24.18 - 2.131 * 4.32 / \sqrt{16} = 21.87$ to $24.18 + 2.131 * 4.32 / \sqrt{16} = 26.48$

A group of 26 rats was selected for a study to test a new drug. Each rat's heart rate was measured prior to receiving that drug and again 2 hours later. The sample mean drop in blood pressure between the readings was 28.2, and the standard deviation was 10.0. You know from previous experiments that the average drop of blood pressure for all other available drugs is 25.0 in a similar setup. Use this data to test your claim that your drug will deliver a better performance than the previously available drugs. Use $\alpha = 0.05$.

H₀: mean = 25

H₁: mean not equal to 25

$t_0 = (28.2 - 25) / (10.0 / \sqrt{26}) = 1.61$

That t_0 value is not large enough (I can tell by now without looking up the t_a) so the test is inconclusive.

The caffeine content of a random sample of 90 cups of black coffee dispensed by a new machine is measured. The mean and standard deviation for the sample are 110 mg and 6.1 mg, respectively.

- a) Compute a 90% confidence interval for the true population mean caffeine content per cup dispensed by the machine.

Whatever the answer might be ☺

- c) If you would compute a 99% confidence interval for the true population, would it be wider or narrower than the 90% confidence interval? (You do **not** actually have to compute this interval to answer the question).

A 99% confidence interval is bigger than a 95% one,

- d) Another person selected a random sample of 900 instead of 90 cups, and the mean and standard deviation of this larger sample turned out to be 110mg and 6.1mg as well. That person uses her data to compute a 90% confidence interval. Would the 90% confidence interval for the larger sample size be wider or narrower than the 90% confidence interval for the smaller sample size?

For larger sample size the confidence interval will be wider

To test the research hypothesis that teacher expectation can improve student performance, two groups of 61 students were compared. Teachers of the experimental group were told that their students would show large IQ gains during the test semester, while teachers of the control group were told nothing. At the end of the semester, IQ change scores were calculated with the following results:

	Mean	Standard Deviation	Sample Size
<i>Experimental</i>	16.5	14.2	61
<i>Control</i>	7.0	13.1	61

Test the null hypothesis of no effect on mean IQ change scores against the above research hypothesis.

This looks like there is a significant difference between the means, doesn't it? Let's compute:

H₀: means are the same

H_a: means are not the same

The combined s = 2.47

$z_0 = 16.5 - 7.0 / 2.47 = 3.8$

p will be less than 0.05, so we reject the null hypothesis, i.e. We think that teacher expectations can indeed improve student performance.

Over the past five years the mean time for a warehouse to fill a buyer's order has been 26 minutes. Officials of the company believe that the length of time has increased recently, either due to a change in the work force or due to a change in customer purchasing policies. The processing time (in minutes) was recorded for a random sample of 121 orders processed over the past month. The mean of that sample is $\bar{x} = 28.20$ and the sample standard deviation is $s = 11.44$. Does this data present sufficient evidence to indicate that the mean time to fill an order has increased, if your error is supposed to be no larger than 0.05? Justify your argument by setting up all four components of a statistical test.

H₀: mean = 26

H_a: mean not equals to 26

$z_0 = (28.2 - 26) / (s / \sqrt{121}) = 2.122$

This will be close, so we compute $p = 2 * P(z > 2.122) = 2 * 0.0174 = 0.0348$, which is less than 0.5, so we reject the null Hypothesis (and accept the alternative). Thus there is sufficient evidence to indicate that the mean order has increased indeed.

A poll of 100 US congress people was taken to determine their opinions concerning a bill to raise the ceiling on the national debt. Each congressperson was then classified according to political party affiliation and opinion on the policy. The results are summarized below. Since the vote is close one would like to find out whether congress voted along party lines or not. Therefore, please test the null hypothesis that these two classifications are independent of one another, against the alternative hypothesis that they are not. Use a level of significance of 0.05

	<i>approve of bill</i>	<i>do not approve of bill</i>	<i>non opinion yet</i>	<i>Total</i>
<i>Republican</i>	28 (22.09)	14 (19.74)	5 (5.17)	47
<i>Democrat</i>	19 (24.91)	28 (22.26)	6 (5.83)	53
<i>Total</i>	47	42	11	100

StatCrunch computes the Chi-Square distribution as follows:

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	6.143 ^a	2	.046
Likelihood Ratio	6.222	2	.045
N of Valid Cases	100		

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 5.17.

This is a Chi-Square test of independence.

H₀: the two variables are not related (are independent)

H₁: the two variables are related

chi_square =6.143 (according to StatCrunch)

p = 0.046, which is smaller than 0.05 so we reject H₀, which means that the variables are related.

We are interested in which person people would have voted for, if they had voted, in 2004. In particular, we want to know if the majority would have voted for or against Georg Bush. We use our GSS data and define a proportion variable to mean 1 if a person would have voted for Bush, and 0 if not. With the help of StatCrunch we conduct a test for propopriion $P_i = 0.5$ and find the following output:

Hypothesis test results:

Outcomes in : WOULD HAVE VOTED FOR IN 2004 = Bush

Success : 1

p : proportion of successes

H₀ : p = 0.5

H_A : p ≠ 0.5

Variable	Count	Total	Sample Prop.	Std. Err.	Z-Stat	P-value
WOULD HAVE VOTED FOR IN 2004 = Bush	195	629	0.3100159	0.019936306	-9.529554	<0.0001

What is your conclusion?

This is a proportion problem. We have declared, arbitrarily, that a vote for Bush is considered a “success”. According to the computation performed by StatCrunch, the p value comes out smaller than 0.0001, and in particular smaller than 0.5. Thus, we reject the null hypothesis and accept the alternative. That means that we reject the idea that people are evenly split into “for” Bush and “against” Bush and therefore say that people have a statistically significant preference. Since the proportion for Bush was less than 0.5, the people’s preference was to vote against Bush.

For the same setup as in the previous question, we have used StatCrunch to compute the confidence interval for P_i , the probability of success. We find:

95% confidence interval results:

Outcomes in : WOULD HAVE VOTED FOR IN 2004 = Bush

Success : 1

p : proportion of successes

Method: Standard-Wald

Variable	Count	Total	Sample Prop.	Std. Err.	L. Limit	U. Limit
WOULD HAVE VOTED FOR IN 2004 = Bush	195	629	0.3100159	0.018441074	0.27387205	0.34615973

What does this mean and how does it connect to your result in the previous question.

This means that the unknown probability of success, i.e. voting for Bush, is estimated to be between 27.4% and 34.6%, with 95% certainty. This estimate does not include 0.5, so we again think – in fact, are 95% certain – that the majority would vote against Bush.

We suspect a coin to be not fair. Suppose we flip that coin 200 times and we come up with 94 heads, 106 tails. Based on this evidence, do you think the coin is unfair?

Another proportion problem. First, we define, arbitrarily, that “heads” is a success. Next, our test looks like this:

H₀: $\pi = 0.5$ (which would mean, incidentally, that the coin is fair)

H₁: π not equal to 0.5 (which would mean an unfair coin)

$$z_0 = (94/200 - 0.5) / \sqrt{(94/200 * 106/200 / 200)} = (0.47 - 0.5) / \sqrt{(0.47 * 0.53 / 200)} = -0.03 / 0.0352 = -0.85$$

$$p = 2 * P(z > |-0.85|) = 2 * P(z > 0.85) = 2 * 0.1977 > 0.05$$

Thus, our test is inconclusive, so as far as we can tell, we're *not* call this an unfair coin.

We conduct a survey to ask people if they are for or against Hydraulic fracturing in a particular county. The survey asked 265 people, 116 came out for the practice, 149 against. Compute a 95% confidence interval for the probability of voting for hydraulic fracturing. If you were to advise a congress person to represent her district accurately, would you advise her to vote for or against the practice?

One last proportion problem: say that success means voting against the practice. Then we have:

H₀ : $\pi = 0.5$

H₁: π not equal to 0.5

$$z_0 = (149/265 - 0.5) / \sqrt{(149/265 * 116/265 / 265)} = (0.56 - 0.5) / \sqrt{(0.56 * 0.44 / 265)} = 0.06 / 0.0305 = 1.9677$$

$p = 2 * P(z > 1.9677) = 2 * 0.0250 = 0.05$, which is just borderline. But since the number we use to lookup is 1.96 whereas the true number $z_0 = 1.9677$, the probability will be just a tad below 0.0250, thus the p-value would be just a tiny bit less than 0.05. Therefore we reject the null hypothesis and accept the alternative, i.e. people are not evenly split but are leaning towards one option. Since our $\pi = 0.56$, we conclude that they are leaning towards being “against” hydrofracking”. Thus, I would advise the congress person to vote against the practice.