

EFFECTS OF ADDITIVE NOISE ON THE THROUGHPUT OF CDMA DATA COMMUNICATIONS

Penina Orenstein, David Goodman, Zory Marantz and Virgilio Rodriguez
Department of Electrical and Computer Engineering¹
Polytechnic University
Brooklyn, NY, USA
porenste@poly.edu

Abstract—We analyze the optimum transmitter power levels and the optimum number of active terminals sending data to a CDMA base station. The objective is to maximize the aggregate throughput of the base station. We find that in the presence of additive noise, received power balancing is suboptimal mathematically. We consider N terminals transmitting at the same data rate, with the power of the most distant terminal (terminal N), fixed at its maximum value, and the power of the other $N-1$ terminals varying. We conclude that the aggregate throughput at the base station is maximized when the receiver powers for the $N-1$ terminals are equal and larger than the receiver power of the N -th terminal. This finding reduces the complexity of the analysis to a univariate optimization problem. A numerical analysis indicates the extent to which additive noise reduces the optimum number of active terminals and the maximum base station throughput.

Keywords—power control; radio resources management; power balancing

I. INTRODUCTION

We analyze the throughput of a CDMA base station receiving data from N transmitters, all operating at the same constant bit rate. We consider two resource management issues: transmitter power control and the number of terminals that should be admitted to the system in order to maximize base station throughput.

Early work on uplink CDMA power control focused on telephone communications and determined that to maximize the number of voice communications, all signals should arrive at a base station with equal power [1]. Initial studies of power control for data communications focused on maximizing the utility of each terminal, with utility measured as bits delivered per Joule of radiated energy [2,3]. By

contrast this paper considers maximizing the aggregate throughput of a CDMA base station.

In earlier work on this problem, Ulukus and Greenstein [9] adjust data rates and transmitter power levels in order to maximize network throughput. Lee, Mazumdar, and Shroff [10] adapt data rates and power allocation for the downlink, and provide a sub-optimal algorithmic solution based on pricing. Sung and Wong [11] assume that the terminals' data rates are different but fixed, and maximize a capacity function.

Our recent work [4-7] adjusts the power and rate of each terminal to maximize $\sum_i \beta_i T_i$, the aggregate weighted throughput of a base station. T_i b/s denotes the throughput of terminal i and the weight β_i admits various interpretations, such as priority, utility per bit, or a monetary price paid by the terminal. This research assumes that the number of active terminals is fixed, their data rates are continuous variables to be optimized, and that the system is interference limited (noise is negligible).

In the present paper, we set all the $\beta_i=1$ and take the data rates to be identical and fixed, but we view the number of active terminals as a key variable to be optimized. We also consider additive noise to be non-negligible, which is essential when out-of-cell interference is significant, and included in the noise term. The research reported in [8] dealt specifically with the case when noise and out-of-cell interference are negligible, and found that the transmitter power levels should be controlled to achieve *power balancing*. With power balancing, all signals arrive at the base station with equal power.

By contrast, the results in the present paper pertain to the case when noise and interference from other cells are *not* negligible. We show that with additive noise *power balancing* leads to sub-optimal performance and that when one terminal has a maximum power constraint, the optimum set of transmitter power levels depends on the maximum received SNR of the constrained terminal. Furthermore, we demonstrate that when one terminal has a maximum power constraint, the other terminals should aim for the *same* received power, which depends on the maximum SNR of the constrained terminal.

¹ Supported in part by NYSTAR through the Wireless Internet Center for Advanced Technology (WICAT) at Polytechnic University and by the National Science Foundation under Grant No. 0219822

*To be concise we refer to the combination of noise and inter-cell interference simply as "noise". The analysis does not distinguish the two impairments. It considers only their combined power.

We also find that in order to maximize base station throughput with any power control algorithm, the number of active transmitters, N , should be limited to $N \leq N^*$, where N^* , is a property of the *frame success rate* $f(\gamma)$, the probability that a terminal's data packet is received successfully as a function of γ , the received signal-to-interference-plus-noise ratio (SINR). The specific form of $f(\gamma)$ depends on the details of the CDMA transmission system, including the packet size, modem configuration, channel coding, antennas, and radio propagation conditions. Our analysis applies to a wide class of practical frame success functions, each characterized by a smooth S-shaped curve [4].

The next section presents the CDMA transmission system and a statement of the throughput optimization problem. The analysis in Section III considers the effects of *non-negligible* noise and out-of-cell interference.

II. THE OPTIMIZATION PROBLEM

A data source generates packets of length L bits at each terminal of a CDMA system. A forward error correction encoder, if present and a cyclic redundancy check (CRC) encoder together expand the packet size to M bits. The data rate of the coded packets is R_s b/s. The digital modulator spreads the signal to produce R_c chips/s. The CDMA processing gain is $G=W/R_s$, where W Hz, the system bandwidth, is proportional to R_c . Terminal i also contains a radio modulator and a transmitter radiating P_i watts. The path gain from transmitter i to the base station is h_i and the signal from terminal i arrives at the base station at a received power level of $Q_i=P_i h_i$ watts. The base station also receives noise and out-of-cell interference with a total power of $\sigma^2=W\eta_0$ watts, where η_0 is the one-sided power spectral density of white noise. The base station has N receivers, each containing a demodulator, a correlator for despreading the received signal, and a cyclic redundancy check decoder. Each receiver also contains a channel decoder if the transmitter includes forward error correction.

In our analysis, the details of the transmission system are embodied in a mathematical function $f(\gamma)$, the probability that a packet arrives without errors at the CRC decoder. The dependent variable γ , is the received SINR. For terminal i ,

$$\gamma_i = G \frac{P_i h_i}{\sum_{j=1, j \neq i}^N P_j h_j + \sigma^2} = G \frac{Q_i}{\sum_{j=1, j \neq i}^N Q_j + \sigma^2} \quad (1)$$

Acknowledgment messages from the receiver inform the transmitter of errors detected at the CRC decoder that have not been corrected by the channel decoder. The transmitter employs selective-repeat retransmission of packets received in error.

Our earlier study [8] assumes that intra-cell interference dominates the total distortion and study system performance when $\sigma^2=0$. When $\sigma^2>0$, we define the signal-to-noise ratio of receiver i as $s_i=Q_i/\sigma^2$ and rewrite Equation (1) as

$$\gamma_i = G \frac{s_i}{\sum_{j=1, j \neq i}^N s_j + 1} \quad (2)$$

In cases of practical interest $f(\gamma)$ is a continuous, increasing S-shaped function of γ , with $f(0)=2^{-M} \approx 0$ and $f(\infty)=1$ [4].

If the probability of undetected errors at the CRC decoder is negligible, the throughput of signal i , defined as the number of information bits per second received without error, is:

$$T_i = \frac{L}{M} R_s f(\gamma_i) \quad \text{b/s}, \quad (3)$$

The aggregate throughput, T_{total} , is the sum of the N individual throughput measures in Equation (3). Assuming that L , M , and R_s are system constants, we analyze the normalized throughput of N simultaneous transmitters as U_N defined as

$$U_N = \frac{M}{LR_s} T_{total} = \sum_{i=1}^N f(\gamma_i) \quad (4)$$

U_N is dimensionless and bounded by $0 \leq U_N \leq N$.

The aim of our optimization study is to find the transmitter power levels, P_i , that maximize U_N . We then examine the maximum throughput as a function of N in order to find the number of simultaneous transmitters that result in the highest normalized throughput. To find the optimum transmitter power levels, it is convenient mathematically to maximize Equation (4) with respect to the received powers Q_1, Q_2, \dots, Q_N . To do so, we differentiate Equation (4) with respect to each of the received power levels Q_i . We then examine the N derivatives under the power balancing condition $Q_i=Q$ for $i=1, 2, \dots, N$. Under this condition, all of the derivatives are equal. They have the following properties.

$$\left. \frac{\partial U_N}{\partial Q_i} \right|_{Q_i=Q} = 0; \quad \sigma^2 = 0; \quad \left. \frac{\partial U_N}{\partial Q_i} \right|_{Q_i=Q} > 0; \quad \sigma^2 > 0 \quad (5)$$

These formulas indicate that when performance is limited by intra-cell interference ($\sigma^2=0$), it is possible that maximum throughput occurs when all signals arrive at the base station at the same power level. The optimization problem is more complex when $\sigma^2>0$.

Our prior work [8] shows that in the absence of additive noise, U_N is maximized when all signals have the same SINR $\gamma=G/(N-1) \approx \gamma^*$, where γ^* , the preferred SINR, is a property of the frame success function $f(\gamma)$. Therefore, with $\sigma^2=0$, the optimum number of active terminals is an integer near $1+G/\gamma^*$.

III. MAXIMUM THROUGHPUT WITH NOISE PRESENT

In this paper we take into account the effects of additive noise and interference from other cells. The total power in these impairments is $\sigma^2=W\eta_0$ watts (and we refer to them together as "noise" to be concise). The noise appears at the receiver as an additional signal that does not contribute to the overall throughput. The system has to use some of its

power and bandwidth resources to overcome the effects of the noise.

The effects of noise depend on the power limits of practical terminals. With unlimited power, we would increase all the received powers Q_i indefinitely until the effect of the noise is negligible. To account for the power limits, let $P_{i,max}$ denote the power of the strongest possible signal transmitted by terminal i and $Q_{i,max}=P_{i,max}h_i$, the power of the corresponding received signal. The maximum signal-to-noise ratio of terminal i is $s_{i,max}=Q_{i,max}/\sigma^2$. In our analysis, we order the labels of the terminals such that $Q_{1,max}\geq Q_{2,max}\geq\dots\geq Q_{N,max}$. In many situations this ordering implies that terminal 1 is closest to the base station and terminal N is most distant.

A. Two terminals

With $Q_{2,max}\leq Q_{1,max}$, it is reasonable to assume that when terminal 2 is admitted to the system, it transmits with maximum power to achieve $Q_2=Q_{2,max}$. In this case, Equation (4) becomes

$$U_2 = f\left(G\frac{Q_1}{Q_{2,max}+\sigma^2}\right) + f\left(G\frac{Q_{2,max}}{Q_1+\sigma^2}\right) \quad (6)$$

If we introduce the normalized quantity $z=s_1/s_{2,max}$ and for conciseness we set $\rho=1/s_{2,max}$, then we can rewrite equation (6) as

$$U_2 = f\left(\frac{Gz}{1+\rho}\right) + f\left(\frac{G}{z+\rho}\right), \quad s_i = Q_i / \sigma^2 \quad (7)$$

The first-order condition for optimality can be expressed as

$$\frac{\partial U_2}{\partial z} = \frac{G}{1+\rho} f'\left(\frac{Gz}{1+\rho}\right) - \frac{G}{(z+\rho)^2} f'\left(\frac{G}{z+\rho}\right) = 0 \quad (8)$$

Using the substitutions

$$\gamma_1 = \frac{Gz}{1+\rho}, \quad \gamma_2 = \frac{G}{z+\rho}, \quad (9)$$

we obtain the first-order condition for optimality:

$$\frac{\gamma_1 f'(\gamma_1)}{\gamma_2 f'(\gamma_2)} = \frac{z}{z+\rho} \quad (10)$$

Equation (5) implies that with $\sigma^2 > 0$, power balancing is suboptimal and that there are transmitter powers with $z > 1$ that produce higher throughput than the throughput obtained when $z=1$.

To establish a necessary condition for a maximum we need to evaluate the sign of $\frac{\partial^2 U_2}{\partial z^2}$ at the critical point. In general,

$$\frac{\partial^2 U_2}{\partial z^2} = \frac{\gamma_1^2 f''(\gamma_1)}{z^2} + \frac{1}{(z+\rho)^2} \frac{\partial}{\partial \gamma_2} (\gamma_2^2 f'(\gamma_2)) \quad (11)$$

If we evaluate equation (11) using the first-order condition in (10), we obtain an expression (12), whose sign determines the nature of the turning point. For a maximum, we require that this expression is negative, i.e.

$$\frac{f''(\gamma_1)}{f'(\gamma_1)^2} + \frac{f''(\gamma_2)}{f'(\gamma_2)^2} + \frac{2}{\gamma_2 f'(\gamma_2)} < 0 \quad (12)$$

Because $f'(\gamma) > 0$ for all γ a sufficient condition for a maximum is that both

- $f''(\gamma_1) < 0$ and $f''(\gamma_2) < 0$
- $\left| \frac{f''(\gamma_1)}{f'(\gamma_1)^2} + \frac{f''(\gamma_2)}{f'(\gamma_2)^2} \right| > \frac{2}{\gamma_2 f'(\gamma_2)}$

For the class of functions considered here, there is a quantity $\tilde{\gamma}$ for which $f''(\gamma) < 0$ when $\gamma > \tilde{\gamma}$. Therefore if $\gamma_2 > \tilde{\gamma}$ and if $z > 1$, it follows that $f''(\gamma_1) < 0$ as required.

For the numerical examples in this paper, we refer to the frame success rate function for the non-coherent frequency shift keying modem and the frame size $M=80$ considered in previous work on power control for wireless data [2-7]. In this case, $f(\gamma) = (1 - 0.5e^{-\gamma/2})^{80}$.

Figure 1 is a numerical example with $\rho=1$ and $G=16.2$, the processing gain that produces $U_2=1$ at $z=1$. The figure shows throughput as a function of z , and graphs for the two sides of Equation (10). It shows that the ‘‘power-balanced’’ solution, $z=1$, is sub-optimal and that equation (10) has solutions at $z=1.41$ and $z=0.65$. Throughput is maximum when $z=1.41$ and a minimum when $z=0.65$. Note that if terminal 1 transmits at a power greater than $1.41P_{2,max}$, then the total throughput will still be better than having just one terminal transmit; however, it is no longer optimal.

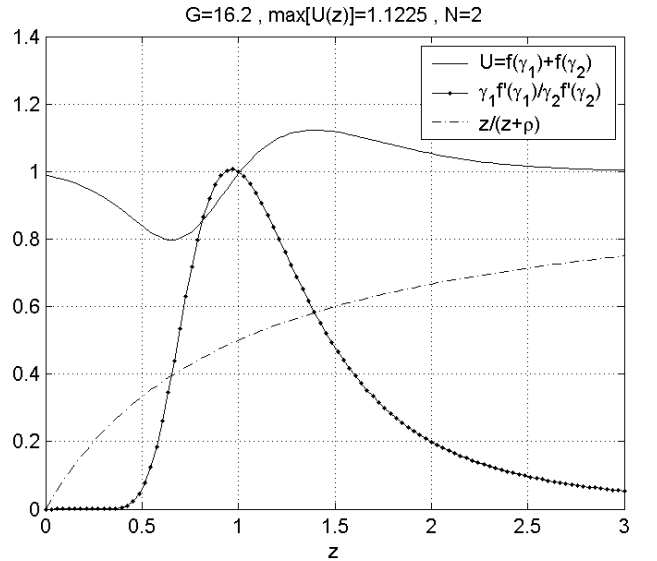


Figure 1: The aggregate throughput U_2 is maximum at power ratio $z=1.41$

B. Arbitrary Number of Terminals

We now move the analysis beyond the case of two terminals. We would like to determine the optimum receive-power vector when there are N terminals, where the power of the weakest terminal (N) is fixed. We begin with a brief analysis for $N=3$ and find that the optimum solution depends on a single variable, which greatly simplifies the problem. We then extend the results to the more general case.

B.1 $N=3$ Terminals

As with $N=2$, we assume that the transmitter power of the weakest terminal is fixed at $P_3=P_{3,max}$, so that $s_3=s_{3,max}$ and we introduce the normalized quantities $z_i=s_i/s_{3,max}$ and $\rho=I/s_{3,max}$. The throughput equation can then be expressed as

$$U_3 = f\left(\frac{Gz_1}{z_2+1+\rho}\right) + f\left(\frac{Gz_2}{z_1+1+\rho}\right) + f\left(\frac{G}{z_1+z_2+\rho}\right). \quad (13)$$

We first evaluate the partial derivatives for terminals 1 and 2 and obtain

$$\left. \begin{aligned} \frac{\partial U_3}{\partial z_1} &= \frac{\gamma_1}{z_1} f'(\gamma_1) - \frac{\gamma_2^2}{Gz_2} f'(\gamma_2) - \frac{\gamma_3^2}{G} f'(\gamma_3) \\ \frac{\partial U_3}{\partial z_2} &= -\frac{\gamma_1^2}{Gz_1} f'(\gamma_1) + \frac{\gamma_2}{z_2} f'(\gamma_2) - \frac{\gamma_3^2}{G} f'(\gamma_3) \end{aligned} \right\} \quad (14)$$

For a critical point we require that both partial derivatives equal zero, from which we obtain the first-order condition

$$\frac{\gamma_1 f'(\gamma_1)(G+\gamma_1)}{\gamma_2 f'(\gamma_2)(G+\gamma_2)} = \frac{z_1}{z_2} \quad (15)$$

Condition (15) can be met if $\gamma_1 = \gamma_2 = \tilde{\gamma} \rightarrow z_1 = z_2$ which implies that a solution exists if the receive power of the two "non-fixed" terminals is equal, yet not necessarily the same as the receive power of the fixed terminal. This solution may not be unique. However, it is useful both for mathematical analysis and for practical implementation.

B.2 Extension to N terminals

The throughput equation for N terminals transmitting to the base station, assuming that $z_i=s_i/s_{N,max}$, for $i=1, \dots, N-1$, and $z_N=s_N/s_{N,max}=I$, can be expressed as

$$U_N = \sum_{i=1}^N f(\gamma_i) = \sum_{i=1}^N f\left(\frac{Gz_i}{\sum_{j=1, j \neq i}^N z_j + \rho}\right) \quad (16)$$

As with the previous analysis we begin by writing $N-1$ the first-order conditions that need to be satisfied simultaneously:

$$\frac{\partial U_N}{\partial z_i} = \frac{\gamma_i}{z_i} f'(\gamma_i) - \sum_{\substack{j=1 \\ j \neq i}}^N \frac{\gamma_j^2}{Gz_j} f'(\gamma_j) \quad i=1, \dots, N-1 \quad (17)$$

This system of equations can be reduced to $N-2$ equations of the form

$$\frac{\gamma_i f'(\gamma_i)(G+\gamma_i)}{\gamma_{i+1} f'(\gamma_{i+1})(G+\gamma_{i+1})} = \frac{z_i}{z_{i+1}} \quad i=1, \dots, N-2 \quad (18)$$

Condition (18) can be met if $\gamma_i = \gamma_{i+1} = \tilde{\gamma}$ for $i=1, \dots, N-2$. It follows that $z_i = z_{i+1}$ for $i=1, \dots, N-2$ from which we conclude that a solution to the first-order conditions is $z_i = z = s/s_{N,max}$ for $i=1, \dots, N-1$. This greatly simplifies our problem since the optimal solution now depends on one power ratio z , rather than N power levels. We can therefore reduce the throughput equation given in equation (16) as follows.

For each $s_i=s$, ($i=1, \dots, N-1$), we write

$$\gamma_i = \gamma = \frac{Gz}{(N-2)z+1+\rho}; \quad \text{and} \quad \gamma_N = \frac{G}{(N-1)z+\rho}$$

Then, the throughput equation can be expressed as

$$U_N(z) = (N-1)f(\gamma) + f(\gamma_N) \quad (19)$$

Using the same approach as before, we can differentiate with respect to z and obtain

$$\frac{\partial U_N}{\partial z} = \frac{((1+\rho)\gamma^2 f'(\gamma))}{Gz^2} - \frac{f'(\gamma_N)\gamma_N^2}{G} \quad (20)$$

which for a critical point yields the first-order condition:

$$\frac{\gamma^2 f'(\gamma)}{\gamma_N^2 f'(\gamma_N)} = \frac{z^2}{1+\rho} \quad (21)$$

If $z=1$, the first order condition cannot be satisfied. Hence, as with $N=2$, we conclude that for a turning point, $z>1$. To classify this point we need to evaluate the sign of the second derivative:

$$\frac{\partial^2 U_N}{\partial z^2} = \frac{\gamma_N^4}{G^2} \left(f''(\gamma) \frac{f'(\gamma_N^2)}{f'(\gamma)^2} + (N-1)f''(\gamma_N) \right) + \quad (22)$$

$$\frac{2f'(\gamma_N)\gamma_N^3}{G^2} \left((N-1) - \frac{\gamma(N-2)}{z\gamma_N} \right)$$

Since $\gamma_N^4, G^2, f'(\gamma_N)^2 > 0$ it follows that the sign of equation (22) is the same as the sign of

$$\underbrace{\frac{f''(\gamma)}{f'(\gamma)^2}}_A + (N-1) \underbrace{\frac{f''(\gamma_N)}{f'(\gamma_N)^2}}_B + \underbrace{\frac{2}{\gamma_N f'(\gamma_N)}}_C \left((N-1) - \frac{\gamma(N-2)}{z\gamma_N} \right) \quad (23)$$

Note that when $N=2$, expressions (23) and (12) are identical. A sufficient set of conditions for a maximum is

- a) $1 < z \leq \frac{\gamma(N-2)}{\gamma_N(N-1)}$,
b) $A + B < 0$,
c) $|A + B| > C$ (24)

One way to satisfy condition (24b) is if both $A < 0$ and $B < 0$. For the class of functions considered here, $f'(\gamma) < 0$ when $\gamma_i > \tilde{\gamma}$. If we assume that both $\gamma_N > \tilde{\gamma}$ and $z > 1$ then it follows that both $f''(\gamma_N) < 0$, $f'''(\gamma) < 0$ as required.

This analysis so far suggests that a local maximum of U_N with respect to z_1, z_2, \dots, z_{N-1} occurs at some value of z such that $z = z_i > z_N$. However, we need to ascertain whether the maximum is at $z = \infty$. A maximum at $z = \infty$ suggests that it might be better to exclude terminal N . The answer depends on ρ . Since we have established that $U_N(z > 1)$ is a maximum, we can say that a sufficient condition for an interior solution is $U_N(\infty) < U_N(1)$, which implies that

$$(N-1)f\left(\frac{G}{N-2}\right) < Nf\left(\frac{G}{N-1+\rho}\right) \quad (25)$$

It follows that the *largest* value of N which satisfies equation (25), say N^* , is the maximum number of active terminals which ensure an interior solution. If $N > N^*$, the aggregate throughput may still be maximized ($z > 1$), however, the power of the “non-fixed” terminals will approach infinity.

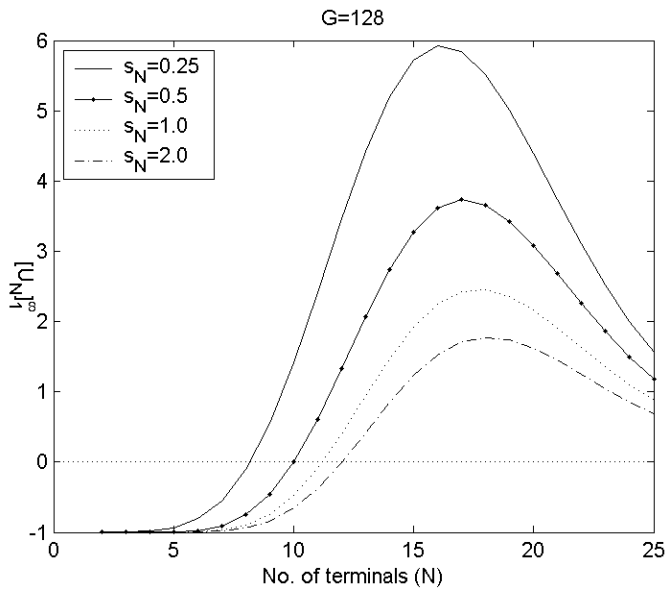


Figure 2: Throughput difference as a function of number of terminals (N). The values of N for which the difference is negative have maximum throughput at $z < \infty$.

Figure 2 shows a numerical example for $G=128$. The vertical axis is $[U_N]_1^{\infty} = U_N|_{z=\infty} - U_N|_{z=1}$. The condition for an interior point (25) is $[U_N]_1^{\infty} < 0$. The graphs indicate the values of N that have an interior solution ($1 < z^* < \infty$). For example, when $s_N=0.5$ ($\rho=2$), $N \leq 10$ satisfies condition (25)

and the maximum throughput occurs at $z < \infty$. The largest value, $N^*=10$ is the maximum number of active terminals which will ensure an interior solution. Using condition (21) we can numerically calculate z as $z=2.24$ which is indeed an interior solution. For $N > N^*$, condition (25) is no longer met and the maximum approaches $z = \infty$. This suggests that in a practical setting, it would be advisable to admit no more than N^* terminals (let $P_i=0$, for $i > N^*$).

It is interesting to note that as $s_N \rightarrow \infty$, the value of N^* approaches 13, which is the optimal number of terminals ($G=128$) when there is no noise: As shown in [8], maximizing the number of active terminals is equivalent to optimizing the function $f(\gamma)/\gamma$ which is maximum at $\gamma^*=10.75$. Thus, $N^* \cong 1 + G/\gamma^* = 12.9$.

CONCLUSIONS

This research considers how to maximize the throughput of a CDMA base station receiving data from N transmitters, all operating at the same constant bit rate. The principal conclusions are that the aggregate throughput at the base station is maximized if $N-1$ transmitters aim for a target SINR that is greater than the maximum SINR of the weakest terminal. Further, the number of active terminals should not exceed N^* , where N^* is the maximum number of terminals that can simultaneously transmit whilst ensuring an interior solution of the first and second order optimality conditions.

REFERENCES

- [1] R.D. Yates, “A framework for uplink power control in cellular radio systems”, IEEE Journal on Selected Areas in Communications, vol. 13, no. 7, pp. 1341–1347, September 1995.
- [2] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, “Efficient power control via pricing in wireless data networks”, IEEE Transactions on Communications, vol. 50, no. 2, pp. 291-303, Feb.2002.
- [3] D. Goodman and N. Mandayam, “Network assisted power control for wireless data”, *Mobile Networks and Applications*, vol. 6, no. 5, pp. 409-418, Sept, 2001.
- [4] V. Rodriguez, “Robust modeling and analysis for wireless data resource management” IEEE WCNC, Vol. 2, pp. 717 -22, March 2003
- [5] V Rodriguez and D.J. Goodman “Prioritized throughput maximization via rate and power control for 3G CDMA: the 2 terminal scenario”, Proc. of 40th Allerton Conf. on Comm., Control and Comp., Oct. 2002
- [6] V. Rodriguez, and D.J. Goodman, “Power and data rate assignment for maximal weighted throughput in 3G CDMA” IEEE WCNC, vol. 1, pp. 525 -31, March 2003
- [7] V Rodriguez, and D.J. Goodman, “Power and Data Rate Assignment for Maximal Weighted Throughput: A Dual-Class 3G CDMA Scenario” IEEE ICC, vol. 1, pp. 397 -401, May 2003
- [8] D.J. Goodman, Z. Marantz, P. Orenstein, and V. Rodriguez, “Maximizing the throughput of CDMA data communications”, IEEE VTC, Orlando FL, Oct 6-9, 2003 (to appear)
- [9] S. Ulukus. and L.J. Greenstein “Throughput maximization in CDMA uplinks using adaptive spreading and power control” IEEE ISSSTA, vol. 2, pp: 565 -569, 2000
- [10] J.W. Lee, R.R. Mazumdar and N.B. Shroff, “Joint power and data rate allocation for the downlink in multi-class CDMA wireless networks”, Proc. of 40th Allerton Conf. on Comm., Control and Comp., Oct. 2002.
- [11] C. W. Sung and W. S. Wong, “Power control and rate management for wireless multimedia CDMA systems”, IEEE Trans. Commun., vol. 49, no. 7, pp. 1215-26, July 2002.

