Fractal structure of traffic jam images

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Abstract The development of area-wide traffic jams over an idealised network can be analysed as a spatial phenomenon. The traffic queues branch out to form a structure with regular geometric features, superimposed on the network of streets through which the vehicles are attempting to pass. The structure of the developed traffic jam appears to have features in common with those of fractal objects.

This paper draws on research on the subject of "Controlling Area-wide Traffic Congestion" being conducted at Middlesex University. The need for an overall conceptual approach has led to the consideration of the suitability of fractal models for describing the form of the traffic jam image. In particular, we have extensively examined the similarities and differences between the vehicle simulation model and the DLA (Diffusion Limited Aggregation) Model.

By extending the network size by five binary orders of magnitude, (ie 16..512), we have been able to assess the Hausdorff dimension of the image using automatic box counting methods. Our results have enabled logarithmic plots to be drawn, with up to nine points represented, each showing approximately linear form. The Hausdorff dimension is estimated to be about 1.7.

Our results have highlighted the fact that an increase in the network size suggests an increase in the Hausdorff dimension, whose numerical value may be dependent on the outline of the final image. The dimension of interior regions of the image appears to be about 1.9.

1 Introduction

The development of area-wide traffic jams over an idealised network can be analyzed as a spatial phenomenon. The traffic queues branch out to form a structure possessing near-regular geometric features, superimposed on the network of streets through which the vehicles are attempting to pass. The structure of the traffic jam appears to have features in common with fractal objects.

The objectives of the research project include deepening the understanding of the formation of traffic jams as well as controlling their dispersal. Earlier research [1] has shown that the growth of a traffic jam can be seen as a branching process which resembles a fractal-like structure. This and the need for an overall conceptual approach has led to the consideration of the suitability of fractal-based growth models for describing the form of the traffic-jam image.

2 A mathematical model for traffic-jam growth

A traffic jam involves a process of aggregation. A vehicle will travel from an origin towards a particular destination. If an obstruction is encountered, the vehicle will be forced to wait until it can proceed. During this time more vehicles will arrive in the road system, some will have avoided the obstructed area and others may not. Queues will begin to form with one vehicle waiting behind another and, depending on the severity of the obstruction, a structure of aggregating vehicles will begin to emerge. This accumulative nature of a traffic jam has enabled us to select a modified version of the mathematical model [2] called DLA over other fractal models.

2.1 Principles of DLA

DLA is a process which generates a single cluster of particles over a network of sites. The network is often referred to as a lattice, this being a regular arrangement of points which fills a space. The model's name suggests a pattern whereby simple particles cling to a self-evolving structure. The sticking is determined by a set of rules. Thus the *aggregation* of particles can be described as a *diffusive* process governed by specific laws which *limit* the development of the cluster over the lattice.

In order to apply the principles of DLA to this piece of research, it has been necessary to relax a number of the assumptions associated with the pure DLA model [3]. The description of the general DLA process refers to this modified version. The *modified* DLA process described in this section acts as a starting point for the representation of traffic on a network. A detailed DLA-type traffic model is presented in the next section.

2.2 The modified DLA mechanism

A circular, bounded region is first selected. The region encloses a regular square lattice. The radius of the bounded region is fixed according to the size of the lattice. A seed particle is fixed somewhere near the centre of the lattice. Before initiating the process it is necessary to select the amount of time required for the growth of a particular cluster. The time element can be viewed as a threshold which, once reached, will force the growth process to terminate. This limit is then divided into sequentially numbered time-slices of equal length. During each time-slice new particles are launched from the perimeter of the bounded region. The number of particles included in a launch follows a Poisson arrival process with a fixed parameter μ. When a launch occurs, the particles begin random walks on the lattice.

Particle States

During each step of a particle's random walk, the particle can assume one of the three following states:

- Fixed Particle, which ceases to move with respect to time,
- Transient Particle, which continues to move with time,
- Destroyed Particle, which is lost to the system.

Particle Movements

A particle can move one square at a time in any of four directions. Each move involves three possible outcomes, as follows

- A particle moves outside the circular bounded region, whereupon it is destroyed.
- A particle approaches the existing cluster. If the particle is within neighbourhood (one-step) of a *fixed* particle, the moving particle gains a *fixed* state and, as a result, its walk is terminated and the cluster is extended.
- A particle advances towards a *transient* (non-fixed) particle. If the particle is within the neighbourhood of a *transient* particle, the movement of the particle will be suspended temporarily and resumed in a subsequent time-slice.

During a time-slice the status of particles occupying sites on the lattice may be be either fixed, destroyed or transient.

Whenever a particle is engaged in a move, the state of the particle is either maintained or updated.

As the particles are dealt with in sequential fashion, the problem of achieving a continuous pattern of movement has been encountered. The effects of this problem have been minimised by selecting relatively short time slices, resulting in a smooth, approximately continuous process.

2.3 DLA-key features

The essential features of the DLA growth process can be summarised thus.

- Lattice of sites.
- Seed particle (with fixed status) planted at the centre of the lattice.
- Growth process of a cluster of particles emanating from the seed.
- Particles released from the edge of a pre-defined region adhere to the law of conservation of particles.
- The resultant image comprises a graphical structure constructed from all fixed particles on the lattice. The image is a connected graph.

3 The traffic simulation model

The evolution of traffic queues over a road network can be viewed as a composite system governed by many interactions. An ideal model is used to focus attention on the features of congestion growth that are not geographically dependent. The simulation model has been developed using the PASCAL programming language for IBM 386 PC-compatible microcomputers.

3.1 The environment

The simple DLA environment has been replaced by a series of one-way streets intersecting at right angles. The system of streets forms a grid-like structure typical of town centres, New York's busy Manhattan being a particular example. A sequence of alternating sources and sinks has been constructed at the edges of the square grid rather than employing a circular bounded region, described in the analytic model. An obstruction is placed near the centre of the network.

Arrival of vehicles at the sources is determined by a Poisson model, reflecting the demand for access to the network of roads. The expected mean of the demand at each access point is described by μ, the model's parameter. A fixed value for μ is used to help create an isotropic flow [1] of vehicles through the system as a whole.

Contrary to the DLA model described in the previous section, the movement of vehicles within a road network is not a random walk. This means that the rules governing vehicular motion appear to be a little more complicated.

[1] See Section 4.

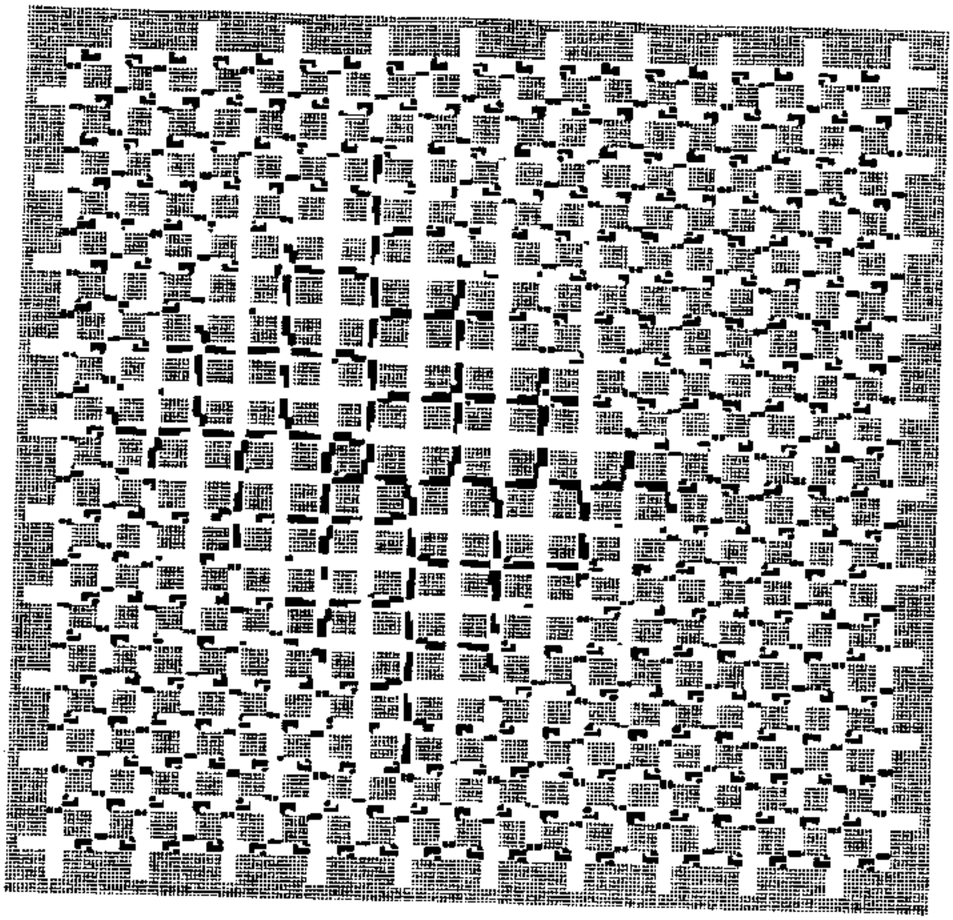

Figure 1. A DLA type traffic jam

The mechanism of vehicle movement, known as the *release mechanism*, mimics simple traffic control. Vehicles are partially segregated into traffic lanes, and the release mechanism allows vehicles to progress subject to vehicle space downstream, capacity constraints at junctions and interactions between queueing vehicles. Vehicles can either maintain their ahead direction or turn in prescribed directions. The propensity of a vehicle to turn is controlled by a fixed probability value which is the same for each intersection. The proportion of vehicles turning is determined at the time of release and is fixed subject to simple numerical rounding.

It is important to note that a stream of random arrivals is introduced to the road network via a stochastic process which only operates at the entry points. The release mechanism works deterministically on the quantity of vehicles that have gained access to the system, and the stochastic behaviour of the model occurs solely at the network's access points.

Instead of considering the movement of individual vehicles within the network, vehicles are assembled into queues, throughout the system, whose states are continually being reviewed.

3.2 Possible states

In keeping with DLA, the three fundamental 'state' categories remain unchanged. However, instead of considering the state of the individual vehicle, we have defined the state of the traffic queues according to the links of the network on which they are located. A link is that portion of a one-way street, between junctions, which contains queues of vehicles characterised by their turn status and queueing discipline (either segregated or mixed queue). In keeping with DLA, links can be *fixed, transient* or *destroyed.* The *destroyed* category can be represented by the links situated at the exit points of the network. These links will be referred to as *sinks* due to the fact that they absorb vehicles leaving the system. *Transient* links are those links which, as the simulation progresses, allow temporary storage and movement of vehicles, and may include the source links whereby vehicles are introduced to the network. A *fixed* link is one where the capacity to accommodate new vehicles is zero and consequently all vehicles in the link are fixed. Our terminology for the *fixed* state is *blocked link,* and the traffic jam is conceived as the aggregation of *blocked links* which form a connected graph. (The simulation preserves *yellow boxes* at junctions which are unavailable for waiting traffic, and hence blocked links are topologically connected.) Because the graph is connected, the vehicles in the blocked link are permanently stationary and conform to the *fixed* status described in the analytical model.

3.3 Vehicular movement

Vehicle queues can only progress in two possible directions, these being *ahead* or *turning,* as described above. Each move can lead to one of three possible outcomes, as follows

- Vehicles can be assigned to one of the sinks, in which case they are lost to the system at the end of the time-slice.
- Vehicles can be assigned to a link with a *transient* state and its queues will be updated, reflecting the movement of vehicles away downstream. Although temporarily at rest, the vehicles would normally be moved on during the next time-slice, though some vehicles may be retained for more than one cycle of the simulation.
- During the assignment process the downstream link may become full. This means that there is no available space for the temporary placement of extra queueing vehicles. Under these circumstances, the link

acquires the *blocked* status, and further movement of vehicles is not allowed. As a result, existing vehicles cannot move out of the link and, in addition, future vehicles will not gain access into the link.

The *blocked* link will increase the extent of the traffic jam, and will be permanent, due to the connected nature of the graph and permanence of the obstruction.

The movement of vehicles, via the release procedure for links, is implemented in two phases. All the east-west movements are processed in the first phase, whilst the north-south movements are realised in the second phase. This encourages a slight asymmetry in transient streets due to the relative position of the image in relation to the simulation time slice. This asymmetry is reflected in slightly larger queues on the east-west road system in comparison with the north-south counterpart. This can be seen in Figure 1. To ensure a regular growth pattern, the model assumes that a steady state flow is in operation before the traffic-jam growth procedure is initiated. A description of the implementation of the steady state flow follows in the next section.

3.4 Simulation model—key features

The essential features of the traffic queues simulation model can be summarised thus.

- Rectangular Grid of road-links and nodes.
- Obstruction (with fixed state) planted at centre of the network.
- Self-Evolving growth process surrounding the core of the obstruction.
- All vehicles released from the edge of the grid are accounted for.
- The resultant image comprises a graphical structure constructed from all fixed streets on the network. The image constitutes a connected graph.

4 An isotropic network

The vehicle simulation model described above differs in one major respect from the DLA model in a way not previously discussed. The DLA growth process starts with an empty system, whilst the vehicle simulation envisages a steady, isotropic flow of vehicles, moving in and out of the system. Growth of the traffic jam is initiated simply by the installation of an obstruction.

4.1 Isotropy—implementation

The isotropic state of the road network has been defined in terms of the system's inputs and corresponding outputs. By introducing a RUNUP period (in which no obstruction is present on the network), it is possible to extract some knowledge about the system's approach to stability. The gradual approach of the network to its steady state is depicted in Figure 2.

We have chosen to record the total input to the road network at the beginning of each time-slice and likewise to calculate the total output at the end of each time-slice. The differences between these two parameters form a sequence which can be tested for stationarity using time-series methods.

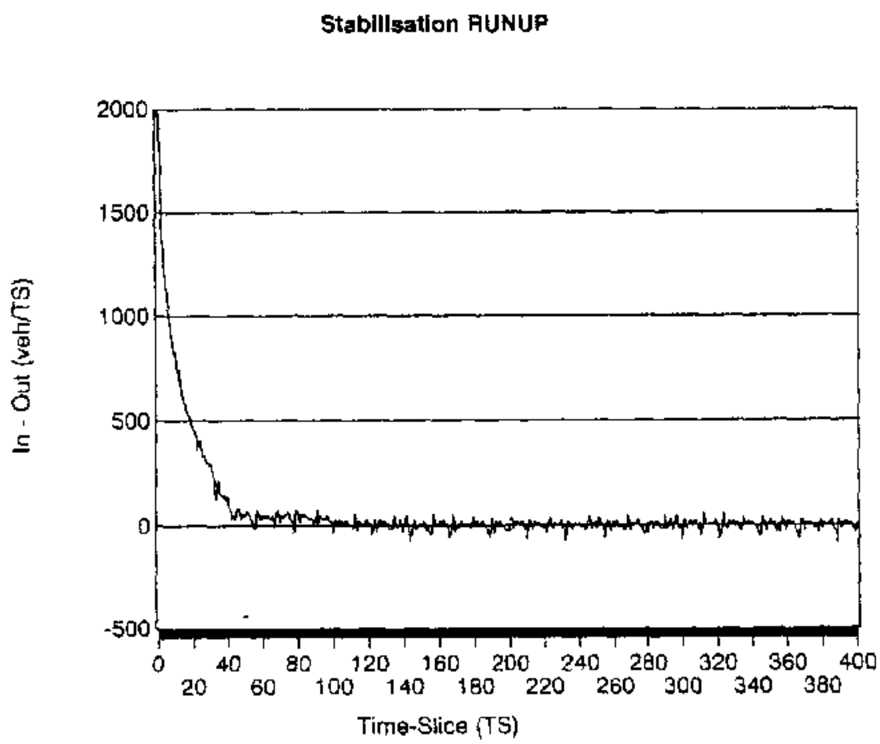

Figure 2. Network's approach to isotropy

Location and Application of Steady-State

If the RUNUP phase involves n time-slices,where n needs to be determined, we can denote the first time-slice by $\tau = \tau_1$ and the last time-slice by $\tau = \tau_n$. The corresponding sequence of numbers, which represent the differences between the system's input and output, are similarly denoted by $y_1 \ldots y_n$.

The sequence $y_1 \ldots y_n$ is partitioned into k samples, each of size ν. The length of ν is chosen empirically, not too long to allow exclusion of initial flows, but not too short to be over-sensitive to local stationarities and turning points. Each sample is considered separately as a time series and is tested for trend and absence of turning points.

If a particular sample $\{ y_j \mid j = (k-1)\nu + 1, \ldots, k\nu \}$ satisfies the conditions for stationarity, then the system has achieved its isotropic state. Hence $n = k\nu$.

Once n has been determined, an obstruction is then introduced near the centre of the grid, a jam evolves from the obstruction and the cycle is repeated until the jam reaches the edge of the grid.

5 A box-counting algorithm—description

The box-counting algorithm [4] has been adapted to assess the Haussdorf dimension of the traffic-jam image. The method of adaptation can be summarised as follows.

- A computer simulation is used to generate a traffic jam image over a network comprising M^2 interconnecting nodes, arranged in the form of an $M \times M$ grid, where $M = 2^k$. (The nodes are connected by streets and in effect act as junctions.) The components of the traffic jam image are precisely those streets which have assumed a *blocked* status.

- A square grid of side L is drawn to enclose the traffic-jam image. The grid is superimposed upon the road network so as to avoid coincidence (Figure 3a), of the road-network structure with the super-imposed grid. The translation of the grid by a constant (Figure 3b), has eliminated the possible ambiguities which could arise in the counting procedure.

- The square grid comprises 2^{2k} square boxes of unit area. (In other words, $k = \log_2 L$.) The grid can be partitioned into $2^{2(k-i)}$ square boxes of side 2^i, where $i = 0 \ldots k$.

  The grid is then partitioned in a number of ways, this procedure being referred to as a *divisions process*. Initially there will be L boxes of side 1, followed by $(L/2)^2$ boxes of side 2 in the second stage, up until the final stage when there will be a single box of side L. At this point, the *divisions process* terminates.

- At each stage in the partitioning procedure, the number of boxes of side $r = 2^i$, denoted by $N(r)$, that intersect the components of the traffic-jam image is recorded. A plot of $\ln(N(r))$ vs $\ln(1/r)$ is

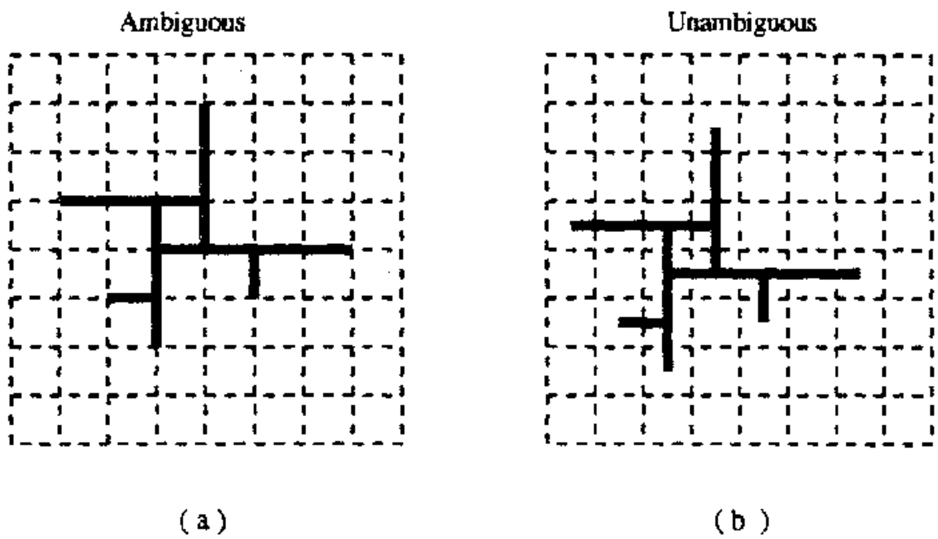

Figure 3. Grid definitions

then obtained. Providing the resultant graph can be described as approximately linear with gradient D, then D can be interpreted as the fractal dimension [5] of the proposed image, since

$$\ln(N(r)) = D * \ln(1/r) + \ln(P),$$

where P is constant, whence

$$N(r) * r^D = P.$$

6 Calculating fractal dimension

The algorithm presented in the previous section can in principle be applied to networks of any size. However, large networks entail a large amount of computer time, and so far we have limited the size of the networks to 512×512 nodes. This, in practice, is very large compared with real road networks. The experiments have been conducted on a Digital Alpha AXP 3000 workstation.

6.1 Shape of traffic jam

The formation process of the traffic-jam structure has been discussed in detail, but little reference has been made to its shape. We have identified

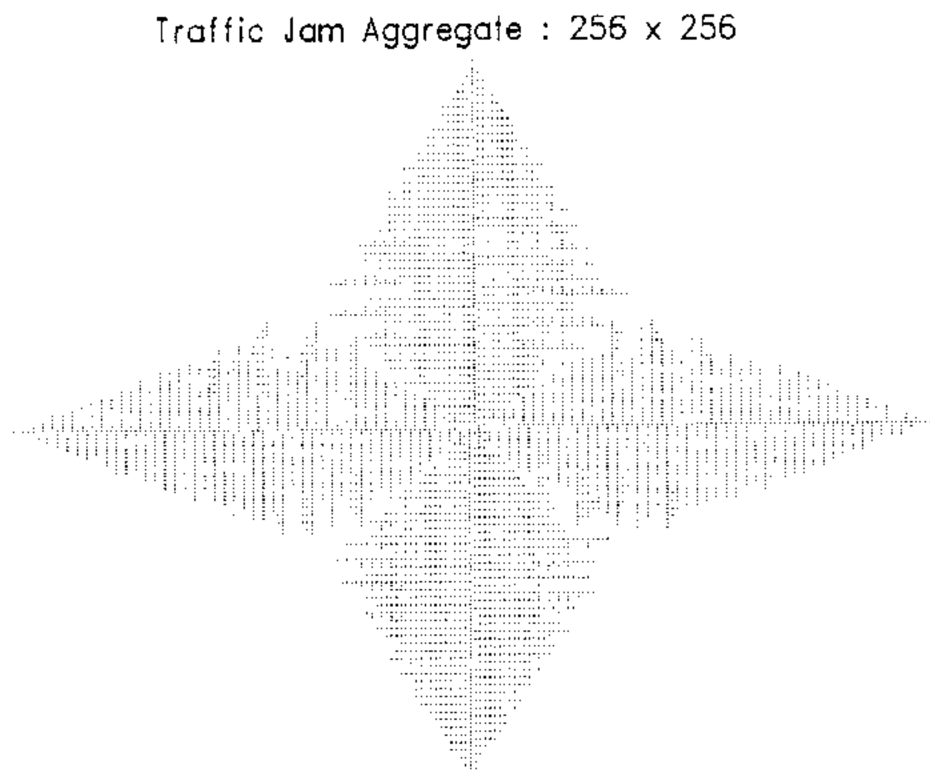

Figure 4. A petalled shaped jam

a pattern of growth which applies to all the sizes of networks considered. The pattern involves four dominant interlocking queues emanating from the source of obstruction. The four primary queues branch out into secondary queues which themselves extend in a self-similar, recursive manner. The resultant cluster displays a characteristic *diamond* shaped envelope as shown in Figure 1. As the network size increases, the *diamond* shape resembles a four petalled flower, (See Figure 4), but still exhibits four distinctive tips. The simulation can be suspended as soon as the tips of the traffic jam extend, but do not intersect the edge of the road-network boundary.

Alternatively, the simulation can be resumed allowing the structure to develop further, until the characteristic diamond or flower has been replaced by a traffic jam which spans the entire square network, to form a *blocked, square network*. Assuming that there is no boundary effect, the resultant image can be viewed as the interior section of a larger, *four-petalled* shaped traffic jam.

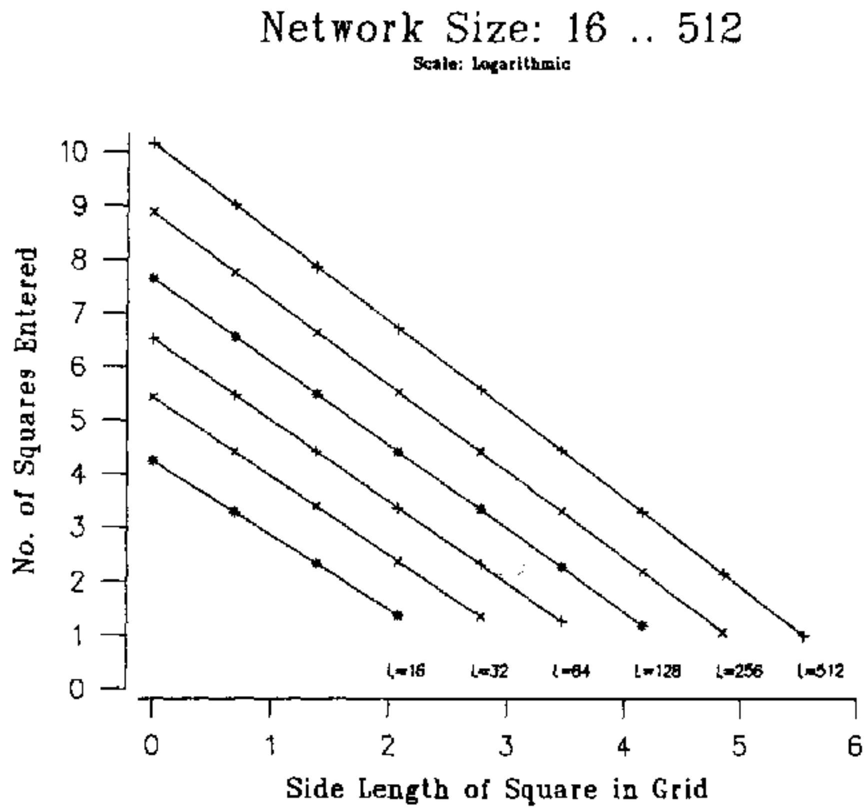

Figure 5. Logarithmic plots: negative slope of line equals fractal dimension

6.2 The experiment

The experiment aims to calculate the Hausdorff dimension of a traffic-jam structure. We have selected two images for consideration, the characteristic diamond as well as the blocked square network, both of which were described in Section 6.1.

First, an $n \times n$ square network is chosen. Once the road network has achieved its steady state, an obstruction is installed at a position which is fixed according to the network size. Denoting this position by the ordered pair $\omega = (x, y)$, such that $(x, y) \in 1 \ldots n$, we have ensured that the ratio $x : y$ remains constant for all the networks considered. The essential parameters required for this stage of the experiment have been summarised in Table 1.

Secondly, the simulation program is executed to produce the required traffic-jam image. The fractal dimension of the image is deduced by applying the techniques described in Section 5. Our results exhibit a strong linear relationship between the log of the divider step and the log of the number of intersections, as shown in Figure 5.

Table 1. Parameter Values used for Generating Traffic Jam

Network Size: n	Steady State: τ_n	Obstacle Position: ω
16	55	(5,8)
32	135	(11,16)
64	300	(22,32)
128	710	(45,64)
256	1650	(90,128)
512	2700	(180,256)

Table 2. Hausdorff Dimension of Traffic Jam

Network Size: n	Fractal Dimension: D_n
16	1.37 ± 0.01
32	1.48 ± 0.03
64	1.52 ± 0.01
128	1.55 ± 0.01
256	1.61 ± 0.00
512	1.65 ± 0.00

6.3 The results

The fractal dimensions, denoted by D_n, for the diamond shaped traffic-jam images are displayed in Table 2. These results are somewhat similar to those obtained by Meakin [6], for a conventional DLA process.

The fractal dimension of the structure that spans the square network has been estimated at 1.90 ± 0.02, $\forall n \in 16 \ldots 512$. This measure will be referred to as D, the subscript having been omitted because the measure applies to all sizes of network considered. D effectively represents the fractal dimension of an interior section of the traffic jam.

7 Geometric interpretation of fractal dimension

The interior fractal dimension D can be explained by considering two similar but not identical situations. In the first, every single link on the road network is blocked, resulting in a totally blocked grid. The second situation, is simply the ***blocked, square network*** described in Section 6.1. The two images differ in one respect only. The second situation entails a system of ***anti-queues***[2] (a series of empty links which complement the traffic jam image), whereas the first does not. By calculating and comparing the *Mass Dimension* [7] of these two images, one can assess the extent to which the

[2] See Section 7.1

presence of ***anti-queues*** affects the interior dimension of a traffic jam. The *Mass Dimension* is denoted by M_d.

7.1 Anti-queues and holes

As seen in Figure 1, a traffic jam is not represented by a totally blocked network. Instead, a feature of a traffic jam is the starvation effect which is caused by the formation of the jam itself. Vehicles that, under unobstructed conditions, could access certain links, are barred entry due to the fact that these links have become surrounded by blocked links. This phenomenon leads to unoccupied areas that are propagated from one link to another in much the same way as queues.

One can view the fress links as *anti-queues* that link together to form an *anti-jam* that is complementary to the jam itself. Besides the *anti-queues* which evolve with the jam, close observation of the graphical images of traffic jams has highlighted the fact that the structure of the jam is not entirely regular. The irregularities manifest themselves as *holes* which appear unpredictably in the structure. The holes are embedded within the traffic jam structure, and close investigation has revealed partially filled links, which are completely surrounded by totally blocked links. This phenomenon has been noticed in internal regions of the jam, which are at quite a distance from the jam boundary. Although they are few in number, it appears that the holes form as a result of the growth of the traffic jam itself.

7.2 Calculation of M_d

The square grid road network of side length L is selected. For the situation in which no *anti-queue* is present, the road network comprises $2L(L+1)$ blocked links, whilst in the second situation the image comprises $2L(1+L/2)$ blocked links. The *Mass dimension* in both cases can conveniently be deduced as $M_d = 2$, the value of the exponent in the limiting form of the two expressions.

This means that the numerical value for M_d is not affected by the presence of anti-queues in the traffic jam image. The application of the *Box-Count Algorithm* to the traffic jam image which includes both anti-queues and spans the entire square network has yielded a dimension of 1.90 ± 0.02. Assuming there is no boundary effect, $D < 2 = M_d$ implies that the structure of the traffic jam includes additional empty space besides the *anti-queues* which form part of the traffic jam image.

In addition, the fact that $D = 1.90 \pm 0.02 < 2$ for any size of square network suggests that the amount of empty space enclosed in a square traffic jam structure scales according to the size of the square network.

7.3 Analysis of D_n

D_n can, in theory, assume values in the range $1 \leq D_n \leq 2$. As $D_n \to 1$, one would deduce that the image approximates a linear structure whereas, if $D_n \to 2$, one could infer that the image approximates a square-like structure. In other words D_n measures the degree to which the traffic jam image fills a two-dimensional space. The crude graphical images of the larger networks suggest that an increase in the network size tends to produce structures which resemble flowers with four distinctly curved petals, whereas the clusters grown on small networks display a characteristic *diamond* shape. The refinement of the shape of the traffic jam may be reflected in the apparently increasing values obtained for D_n, a higher D_n implying a more space-filling traffic-jam structure. In Figure 1 the boundary of the traffic jam can be viewed as approximately step-like, and to a degree, almost linear whereas with Figure 4 the curvy boundary is reflected in a fractal dimension closer to 2 than 1.

8 Conclusions and further work

The research has established that traffic-jam images display fractal characteristics. The images form self-similar structures, in that the growth pattern of traffic queues is replicated across several scales. In addition, the Hausdorff Dimension of the images under consideration does not exceed the Euclidean dimension, indicating that the images represent fractal forms.

A fractal object often involves an underlying growth process, and we have identified aspects of similarity between the DLA growth process and the traffic-jam aggregation mechanism. However, we have highlighted major differences between the formal specification of DLA and the vehicle simulation, such as the movement of traffic queues inside the region of consideration, and the necessity to operate a steady flow of vehicles before the aggregation process can commence.

Exploratory assessments of the Hausdorff Dimension have led to the discovery of irregularities in the form of the traffic-jam image, but more research into the distribution and extent of the holes in the traffic jam structure is required. We envisage developing more representative pictures, particularly for larger networks, which would enable us to assess the possible fractal nature of the holes incorporated in the image. In addition, we would like to consider the density of the jam at various radii of gyration, which would enable us to understand whether holes are likely to appear at particular locations in the structure.

Our research has highlighted the fact that the shape of a traffic jam becomes more intricate as the size of the growth environment increases and has suggested that this refinement of shape may contribute to the reasoning behind the apparently increasing values of D_n. More research is required to determine the implications for the nature of D_n, and we hope to proceed by increasing our field of investigation to include still larger networks. This would enable us to establish the extent to which D_n appears to approach an upper bound, and if possible ascertain this limit.

We would like to direct the modelling work towards the self-controlling nature of fractal-like traffic jams. By identifying factors in the growth environment which contribute to the development of the traffic-jam cluster, it may be possible to inhibit the onset of *gridlock*. This time-oriented gain would enable added flexibility to the successful implementation of Traffic Control Strategies, which we hope to analyse by developing the Traffic Simulation Model.

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