MAXIMIZING THE THROUGHPUT OF CDMA DATA COMMUNICATIONS THROUGH JOINT ADMISSION AND POWER CONTROL

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***Abstract* - We consider N terminals transmitting at the same data rate (G), with the power limits on the N terminals being P_{max}. We assume that the power of the most distant terminal (terminal N) is fixed at its maximum value, P_{max} and arbitrary noise power is σ^2. The path gains of the N terminals are given by h_i, corresponding to the locations of the terminals. The objective is to maximize the aggregate throughput at the base station, in terms of (a) the optimum number of terminals admitted to the system and which ones and (b) the power levels of each admitted terminal. We also describe the maximum throughput as a function of data rate and noise power level.**

Keywords-power control; radio resources management; power balancing, admission control

I. INTRODUCTION

Early work on uplink CDMA power control focused on telephone communications and determined that to maximize the number of voice communications, all signals should arrive at a base station with equal power [1]. Initial studies of power control for data communications focused on maximizing the utility of each terminal, with utility measured as bits delivered per Joule of radiated energy [2,3]. By contrast this paper considers maximizing the aggregate throughput of a CDMA base station.

A different approach to this problem, is described by Ulukus and Greenstein [4] who adjust data rates and transmitter power levels in order to maximize network throughput. Lee, Mazumdar, and Shroff [5] adapt data rates and power allocation for the downlink, and provide a sub-optimal algorithmic solution based on pricing. Sung and Wong [6] assume that the terminals' data rates are different but fixed, and maximize a capacity function.

A related strand of recent work [7-10] adjusts the power and rate of each terminal to maximize $\Sigma_i \beta_i T_i$, the aggregate weighted throughput of a base station. T_i b/s denotes the throughput of terminal i and the weight β_i admits various interpretations, such as priority, utility per bit, or a monetary price paid by the terminal. This research assumes that the number of active terminals is fixed, their data rates are continuous variables to be optimized, and that the system is interference limited (noise is negligible).

In this paper, we set all the β_i=1 and take the data rates to be *identical* and *fixed*, but we view the number of active terminals and their associated power levels as key variables to be optimized. We also consider additive noise to be *non-negligible*, which is essential when out-of-cell interference is significant, and included in the noise term.

Our recent work reported in [11] dealt specifically with the case when noise and out-of-cell interference* are negligible, and found that the transmitter power levels should be controlled to achieve *power balancing*. With power balancing, all signals arrive at the base station with equal power.

By contrast, in [12], we find that with additive noise (a) *power balancing* leads to sub-optimal performance and (b) that when one terminal has a maximum power constraint, the other terminals should aim for the *same* received power, which depends on the maximum SNR of the constrained terminal.

Here, we briefly review the main result from [12] and then extend the analysis to find the number of active

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*To be concise we refer to the combination of noise and inter-cell interference simply as "noise". The analysis does not distinguish the two impairments. It considers only their combined power.

transmitters, N. We show that in order to maximize base station throughput with any power control algorithm, that N should be limited to $N \le N^*$, where N^*, is a function of *background noise* and the *frame success rate* $f(\gamma)$, the probability that a terminal's data packet is received successfully as a function of γ, the received signal-to-interference-plus-noise ratio (SINR).

The specific form of $f(\gamma)$ depends on the details of the CDMA transmission system, including the packet size, modem configuration, channel coding, antennas, and radio propagation conditions. Our analysis applies to a wide class of practical frame success functions, each characterized by a smooth S-shaped curve [4].

The outline of the paper is as follows. We first present the details of the CDMA transmission system and a brief statement of the throughput optimization problem. We then examine the effects of *non-negligible* noise and out-of-cell interference on *both* the optimal number of transmitters and the receive-power ratio which jointly maximize base-station throughput. We also show how noise reduces the number of active terminals and illustrate our results with a numerical example. The implications for admission control are then briefly discussed.

II. THE OPTIMIZATION PROBLEM

A data source generates packets of length L bits at each terminal of a CDMA system. A forward error correction encoder, if present, and a cyclic redundancy check (CRC) encoder together expand the packet size to M bits. The data rate of the coded packets is R_s b/s. The digital modulator spreads the signal at a rate of R_c chips/s. The CDMA processing gain is $G=W/R_s$, where W Hz, the system bandwidth, is proportional to R_c. Terminal i also contains a radio modulator and a transmitter radiating P_i watts. The path gain from transmitter i to the base station is h_i and the signal from terminal i arrives at the base station at a received power level of $Q_i=P_ih_i$ watts. The base station also receives noise and out-of-cell interference with a total power of $\sigma^2=W\eta_0$ watts, where η_0 is the one-sided power spectral density of white noise. The base station has N receivers, each containing a demodulator, a correlator for despreading the received signal, and a cyclic redundancy check decoder. Each receiver also contains a channel decoder if the transmitter includes forward error correction.

In our analysis, the details of the transmission system are embodied in a mathematical function $f(\gamma)$, the probability that a packet arrives without errors at the CRC decoder. The dependent variable γ, is the received SINR. For terminal i,

$$\gamma_i = G\frac{P_ih_i}{\sum_{\substack{j=1\\ j\neq i}}^{N} P_jh_j+\sigma^2} = G\frac{Q_i}{\sum_{\substack{j=1\\ j\neq i}}^{N} Q_j+\sigma^2} \qquad (1)$$

Acknowledgment messages from the receiver inform the transmitter of errors detected at the CRC decoder that have not been corrected by the channel decoder. The transmitter employs selective-repeat retransmission of packets received in error.

Our earlier study [11] assumes that intra-cell interference dominates the total distortion and examines system performance when $\sigma^2=0$. When $\sigma^2>0$, we denote the signal-to-noise ratio of receiver i as $s_i=Q_i/\sigma^2$ and rewrite Equation (1) as

$$\gamma_i = G\frac{s_i}{\sum_{j=1, j\neq i}^{N} s_j+1}. \qquad (2)$$

In cases of practical interest $f(\gamma)$ is a continuous, increasing S-shaped function of γ, with $f(0)=2^{-M}\approx 0$ and $f(\infty)=1$ [3].

If the probability of undetected errors at the CRC decoder is negligible, the throughput of signal i, defined as the number of information bits per second received without error, is:

$$T_i = \frac{L}{M}R_sf(\gamma_i) \text{ b/s}, \qquad (3)$$

The aggregate throughput, T_{total}, is the sum of the N individual throughput measures in Equation (3). Assuming that L, M, and R_s are system constants, we analyze the normalized throughput of N simultaneous transmitters *as* U_N defined as

$$U_N = \frac{M}{LR_s}T_{total} = \sum_{i=1}^{N} f(\gamma_i)\,. \qquad (4)$$

U_N is dimensionless and bounded by $0\le U_N\le N$.

III. FIRST-ORDER NECESSARY CONDITIONS

As stated earlier, this paper deals with the effects of additive noise and interference from other cells. The total power in these impairments is $\sigma^2=W\eta_0$ watts (and we refer to them together as "noise" to be concise). The noise appears at the receiver as an additional signal that does not contribute to the overall throughput. The system has to use some of its power and bandwidth resources to overcome the effects of the noise.

The effects of noise depend on the power limits of practical terminals. With unlimited power, we would increase all the received powers Q_i indefinitely until the effect of the noise is negligible. To account for the power limits, let $P_{i,max}$ denote the power of the strongest possible signal transmitted by terminal i and $Q_{i,max}=P_{i,max}h_i$, the power of the corresponding received signal. The maximum signal-to-noise ratio of terminal i is $s_{i,max}=Q_{i,max}/\sigma^2$. In our analysis, we order the labels of the terminals such that $Q_{1,max}\geq Q_{2,max}\geq\ldots\geq Q_{N,max}$. In many situations this ordering implies that terminal 1 is closest to the base station and terminal N is most distant.

We would like to determine the optimum receive-power vector when there are N terminals, where the receive power of the weakest terminal (N) is fixed.

A.1 Generalized case: N terminals

The throughput equation for N terminals transmitting to the base station, assuming that $z_i=s_i/s_{N,max}$, for $i=1,\ldots,N-1$, $z_N=s_N/s_{N,,max}=1$, and $\rho=1/s_{N,max}$, can be expressed as

$$U_N=\sum_{i=1}^{N} f(\gamma_i)=\sum_{i=1}^{N} f\left(\frac{Gz_i}{\sum\limits_{\substack{j\neq i\\ i=1}}^{N} z_j+\rho}\right) \tag{1}$$

We begin by writing the $N-1$ first-order conditions that need to be satisfied simultaneously:

$$\frac{\partial U_N}{\partial z_i}=\frac{\gamma_i}{z_i}f'(\gamma_i)-\sum_{\substack{j\neq i\\ i=1}}^{N}\frac{\gamma_j^{\ 2}}{Gz_j}f'(\gamma_j)\qquad i=1,..,N-1 \tag{2}$$

This system of equations can be reduced to $N-2$ equations of the form

$$\frac{\gamma_i f'(\gamma_i)(G+\gamma_i)}{\gamma_{i+1}f'(\gamma_{i+1})(G+\gamma_{i+1})}=\frac{z_i}{z_{i+1}}\qquad i=1,..,N-2 \tag{3}$$

By applying a similar analysis to case of $N=3$, see [12,13] for details, we can reduce this expression further and obtain

$$\frac{\gamma_i f'(\gamma_i)(G+\gamma_i)}{\gamma_{i+1}f'(\gamma_{i+1})(G+\gamma_{i+1})}=\frac{\gamma_i(G+\gamma_{i+1})}{\gamma_{i+1}(G+\gamma_i)}\qquad i=1,..,N-2 \tag{4}$$

A solution to equation (4) coincides with $\gamma_i=\gamma_{i+1}=\widetilde{\gamma}$ for $i=1,\ldots,N-2$, from which it follows that $z_i=z_{i+1}$ for $i=1,\ldots,N-2$. We demonstrate the uniqueness of this solution in [13].

We conclude that a solution to the first-order conditions is $z_i=z=s/s_{N,max}$ for $i=1,\ldots,N-1$. This greatly simplifies our problem since the optimal solution now depends on one power ratio z, rather than N power levels. We can therefore reduce the throughput equation given in equation (1) as follows.

For each $s_i=s$, ($i=1,\ldots,N-1$), we write

$$\gamma_i=\gamma=\frac{Gz}{(N-2)z+1+\rho};\quad\text{and}\quad \gamma_N=\frac{G}{(N-1)z+\rho}$$

Then, the throughput equation can be expressed as

$$U_N(z)=(N-1)f(\gamma)+f(\gamma_N) \tag{5}$$

Using the same approach as before, we can differentiate with respect to z and obtain

$$\frac{\partial U_N}{\partial z}=\frac{((1+\rho)\gamma^2 f'(\gamma))}{Gz^2}-\frac{f'(\gamma_N)\gamma_N^{\ 2}}{G} \tag{6}$$

Now equation (6) can be simplified if we perform the following algebraic manipulations. Let

$$(N-2)z+1+\rho=\frac{G}{\gamma_N}+(1-z) \tag{7}$$

and then express γ as

$$\gamma=\frac{Gz}{G/\gamma_N+(1-z)} \tag{8}$$

We can further reduce condition (8) as

$$z=\frac{(G+\gamma_N)\gamma}{(G+\gamma)\gamma_N} \tag{9}$$

Substituting (9) into condition (6) results in

$$\frac{\partial U}{\partial z}=\frac{\gamma_N^{\ 2}}{G}\left((1+\rho)f'(\gamma)\frac{(G+\gamma)^2}{(G+\gamma_N)^2}-f'(\gamma_N)\right) \tag{10}$$

which yields the following critical points at $\frac{\partial U}{\partial z}=0$:

a) $G=\infty$, or
b) $\gamma_N=0$ or
c) $(1+\rho)f'(\gamma)\frac{(G+\gamma)^2}{(G+\gamma_N)^2}=f'(\gamma_N)$

(11)

We can classify each of these critical points by examining the second-order conditions, [13]. In this paper, we summarize the key results as follows: (1) with no noise, (ρ=0), throughput is maximized when all signals are received with equal power ($\gamma=\gamma_N$) providing $N<N^*$. Otherwise some terminals should use $P=0$; (2) with additive noise (ρ>0), equation (11c) is not satisfied with $\gamma=\gamma_N$ and throughput is maximized when (a) one terminal has a maximum power constraint (P_{max}), and (b) the other N-1 terminals aim for the same received power, which depends on the maximum SNR of the constrained terminal.

We would like to determine N*, the optimum number of terminals which can be simultaneously supported when there is additive noise and both (a) and (b) are satisfied.

We begin by defining the *largest* value of N which satisfies equation (5), say N^*, as the maximum number of active terminals which ensure that (a) the throughput is maximized and (b) that the power vector be an interior solution. By this we mean that $\gamma_N \neq 0$, since it is possible to find $N>N^*$, such that the aggregate throughput is maximized ($\gamma>\gamma_N$), however, the power of the "non-fixed" terminals will approach infinity.

A.2 Optimal Number of Active Transmitters N*

If we optimize with respect to N, we obtain

$$\frac{\partial U_N(z)}{\partial N} = f(\gamma) - \frac{(N-1)\gamma^2}{G} f'(\gamma) - \frac{z\gamma_N^{\ 2}}{G} f'(\gamma_N) \tag{12}$$

For a critical point, we need to set equation (12) to zero. We can then substitute the expression obtained in (6), to obtain

$$f(\gamma) = \frac{\gamma^2}{G} f'(\gamma)\left(\frac{(N-1)z+1+\rho}{z}\right) \tag{13}$$

We can reduce equation (13) by writing $\gamma = \dfrac{Gz}{(N-2)z+1+\rho}$ as $\dfrac{Gz}{\gamma} = (N-1)z+1+\rho-z$ which can

then be reduced to

$$\frac{G}{\gamma}+1 = \frac{(N-1)z+1+\rho}{z} \tag{14}$$

All that remains is to simplify equation (13) using (14) which results in

$$\frac{f(\gamma)}{\gamma(1+\gamma/G)} = f'(\gamma) \tag{15}$$

One can therefore solve equation (15) to find the optimal number of active transmitters. This is equivalent to maximizing with respect to N. Further, it can be seen that as G→∞ (equivalently, condition 11a), that maximizing U_N with respect to N is equivalent to maximizing $f(\gamma)/\gamma$. For the class of functions f being considered, $f(x)/x$ has a unique maximum at the point where a line from the origin is tangent to $f(x)$, [4]. We use the notation γ^* for the signal-to-interference ratio that maximizes $f(x)/x$. γ^* is the unique solution to the equation

$$\frac{f(\gamma)}{\gamma} = f'(\gamma) \tag{16}$$

Further, we demonstrated how maximizing $f(\gamma)/\gamma$ was equivalent to maximizing the number of users without noise (ρ=0, $\gamma=\gamma_N$). For the class of function being considered, the unique solution was given by γ*=10.75, and the corresponding optimal number of transmitters was found to be N*=G/γ*+1, [11].

One can view this result as a special case of equation (15). In other words, unbounded G is equivalent to treating noise and out-of-cell interference negligible. Thus, $G=\infty$ is the same as having $\rho=0$ and $\gamma=\gamma_N$ ($z=1$). This is intuitive, since unlimited bandwidth allows as many users as possible to transmit. However, in practical systems, bandwidth is a finite resource and (1) the impact of noise is non-negligible and (2) the effect on the number of transmitters needs to be ascertained.

IV. NUMERICAL RESULTS

So far, we have provided a theoretical framework to explain the effects of noise, both in terms of the optimal number of transmitters as well as the corresponding optimum receive-power vector. In this section, we illustrate the analysis with numerical examples and then discuss the implications for admission control. Throughout, we will assume a fixed value for $Q_{N,max}$ and use this to ascertain z, the ratio of γ to γ_N.

Figure I shows a graphical plot of the left side equation (15) for different values of G. One can observe the "no-noise" case as $\rho=0$, $\gamma=\gamma_N$, (alternatively G=∞). This is the upper bound on the system and corresponds to the first-order condition given by (11a). Figure I also displays f'(γ) as a function of γ. The intersection of this curve with one of the other curves in Figure I occurs at γ=γ*, the optimum signal-to-interference-plus-noise ratio.

For example, when G=16, γ*=12.11, and similarly when G=40, γ*=11.354 and when G=128, γ*=10.94. Note that the critical value of γ* approaches 10.75 as the system bandwidth increases.

We can find N* using the unique value of γ obtained from the solution to equation (15). We shall refer to this value as γ*, the critical SNR for the N*-1 terminals. But, by definition, γ* must satisfy

$$\gamma^* = \frac{Gz}{(N-2)z+1+\rho}$$

It follows that upon dividing through by z ($1<z\leq\infty$) and substituting equation (9) we can obtain an expression for N* as

$$N^* = \frac{G}{\gamma^*} - \frac{(1+\rho)(G+\gamma^*)\gamma_N}{(G+\gamma_N)\gamma^*} + 2 \qquad (17)$$

Note that when ρ=0 and $\gamma=\gamma_N$ expression (17) reduces to $N^* = \frac{G}{\gamma^*}+1$ which was found to be the optimum number of terminals for the "no noise" case, [11]. This condition can be viewed as the upper bound on the system.

It follows that condition (17) is a more general form and can be used to ascertain the optimum number of active transmitters (N*) which can be admitted given the level of background noise and γ* which is determined via equation (15).

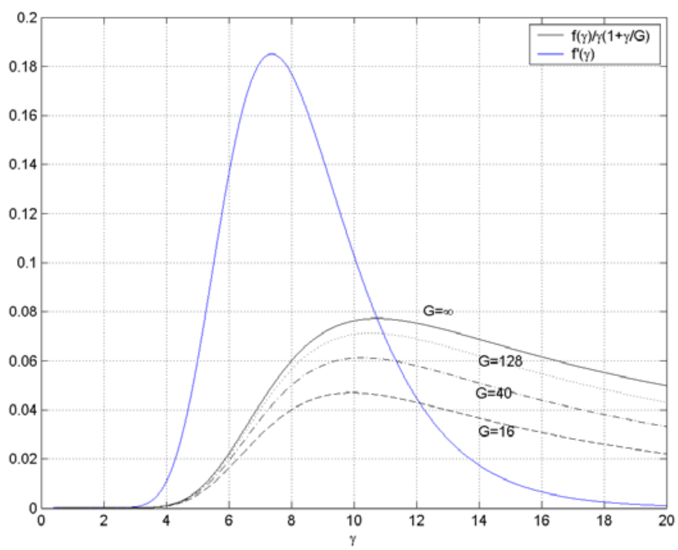

Figure I: Effect of bandwidth on γ*: Finite values of G increase γ* beyond γ*=10.75, the optimal γ* for the "no-noise" case (equivalently G=∞).

Next, we examine the relationship between the sub-optimal solution ($\gamma=\gamma_N$)and the optimal solution found in [12], which occurs when $\gamma>\gamma_N$. In the next example, we show that the sub-optimal solution is a good approximation and that the gains obtained by setting $\gamma>\gamma_N$ do not appear to be significant. We demonstrate this using the numerical example in Figure II.

With ρ=1, N*=12, and the throughput, U_N, is maximum at γ/γ_N= *z=1.176>1*. However, one can observe that $\gamma=\gamma_N$ (*z=1*) is a close approximation. There may be situations (depending on ρ) where this difference might be more significant, however in this example one can see that U_N is nearly the same for $\gamma=\gamma^*=\gamma_N$ as it is for $\gamma>\gamma_N$.

Further, since ρ=1, the maximum number of transmitters is one less than the case without noise and if N>12, there is no solution except at z=∞. The graph also shows that if N<N*, then throughput can be maximized at $\gamma=\gamma_N$ or *z=1*.

Based on this observation, we then plot equation (15) and observe *N**, the optimal number of transmitters as shown in Figure III. If we evaluate equation (17) at $\gamma=\gamma_N$ we obtain a bound on *N**: Thus

$$N^*_{\gamma=\gamma_N} = \frac{G}{\gamma^*}+1-\rho \qquad (18)$$

Assuming that the sub-optimal solution $\gamma=\gamma_N$ is a good approximation, it follows that equation (18) relates the optimal number of terminals *N** to the background noise level. It can be observed that noise acts as *ρ* interfering terminals. An example is shown in Figure III.

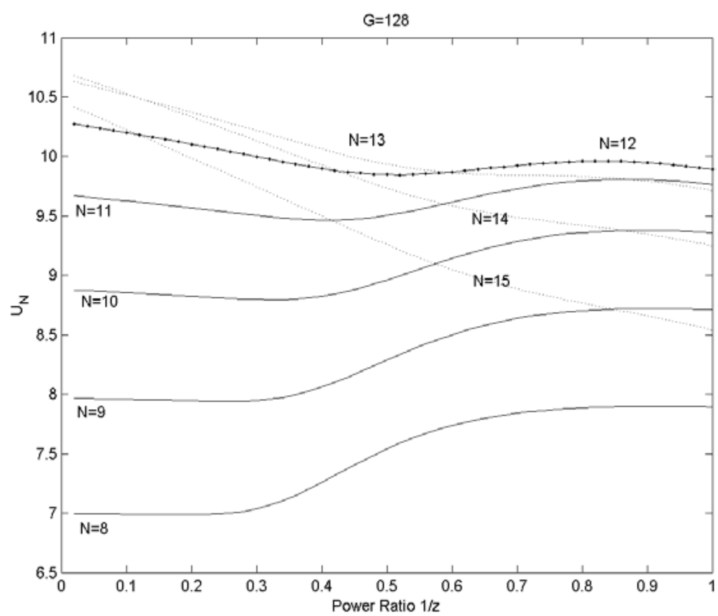

Figure II: With G=128 and ρ=1, we find that U_N is largest when N*=12 and maximized for γ/γ_N=*z=1.176*. At $\gamma=\gamma_N$ *(z=1)*, U_N is close to optimal. Other optimal power ratio allocations are possible for N<N*.

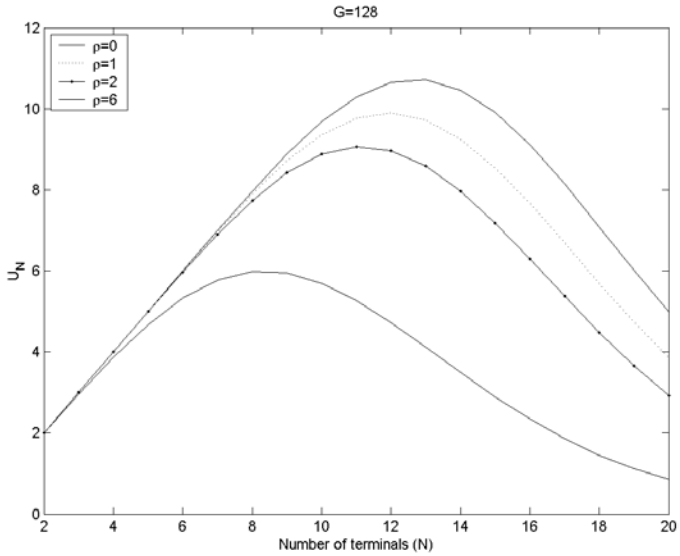

Figure III: The effect of noise on throughput and the optimum number of admitted terminals: when ρ=0 (no noise), N*=13. Further increases in ρ reduce the optimal number of active terminals, N*.

Our results can be used as part of a simple admission control scheme. Given ρ_N, the noise-to-signal ratio of the "weakest" terminal, where $\rho_N > \rho_{N-1} > \ldots > \rho_1$, and the data rate G, we can determine N*, the optimum number of active terminals and their power levels such that the aggregate throughput at the base-station is maximized.

We first apply equation (18) with the pair (N, ρ_N) to find the optimal number of terminals which can be admitted under the near-optimum *power-balancing* assumption ($\gamma = \gamma_N$). We then calculate the corresponding throughput given by equation (15). We then eliminate the "weakest" terminal and apply the algorithm in a recursive manner. When the algorithm terminates, we will have a vector of values for N* and a corresponding vector for U_{N^*}.

The objective is to find a value of N* as close as possible to $\frac{G}{\gamma^*}+1$, the upper bound on the system, which also maximizes U_{N^*}. Under certain physical assumptions, this algorithm will help decide how many terminals to admit from the population and which ones. This analysis is described in detail in [13].

CONCLUSIONS

In this paper, we examined the effects of *non-negligible* noise and out-of-cell interference on *both* the optimal number of transmitters and the receive-power ratio which jointly maximize base-station throughput. We showed that with noise – throughput is not mathematically maximized using power-balancing; however, for practical purposes, power-balancing is nearly optimum.

Further, we tie two strands of research together: the "no-noise" case described in [11] and the equivalent system where "noise" is explicitly considered. We show that the former is a special case of the latter and can be viewed as an upper bound on the system. We also quantify how "noise" reduces the optimal number of active terminals.

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