**Math 4516: Final Exam**

*This is a take-home exam, due on the last day of final’s week. You can submit it in person or via email. You must complete the exam on your own, but you may use any other resource as necessary.*

1. **Taylor Series**
2. Find Taylor series centered at zero for , including the radius of convergence.
3. Prove Lagrange’s version of the remainder for Taylor series.
4. **Lebesgues Outer Measure and Measure**
5. State the definition of a set being Lebesgue measurable. Use that definition and some basic properties of measure to show that every set in ***R*** with outer measure zero is measurable.
6. Show that given any set *A* and any there is an open set *O* such that and . Then use that result to show that for any set *A* there also exists a set *G* such that and . Note that a set is a set if it is the countable intersection of open sets ( for “Durchschnitt”, German for intersection).
7. **Lebesgue Integration**
8. Show that if *f* is bounded and integrable and then
9. Show that every continuous function on *[a, b]* is Lebesgue integrable.
10. **Fourier Series and Fourier Transform**
11. Find the Fourier series of the function . Verify your calculations by plotting the sum of the first, say, 50 terms using Mathematica.
12. Extend the function above to be defined on all of , with outside the interval . Find the Fourier transform for

**5. Metric Spaces**

1. Show that for any metric space we have
2. Consider , i.e. the space of all sequences of numbers such that with the standard metric. Define a function such that . Is *f* 1-1? Is it onto? Is it continuous? Justify your answers. Finally, find for

**BONUS: Littlewood’s Three Principles**

In their presentation, Carmela and Sheba identified *Littlewood’s Three Principles* as

* 1. Every measurable set is “nearly” a finite union of intervals.
  2. Every measurable function is “nearly” continuous.
  3. Every convergent sequence of measurable functions is “nearly” uniformly convergent.

In class, we introduced the named theorems Egorov’s Theorem, Luzin’s Theorem, Lebesgue Dominated Convergence Theorem, Lebesgue Monotone Convergence Theorem, and Fatou’s Lemma. Two of these theorems are equivalent to two of the above principles; who is who? No proof necessary, just match up two of the principles with two of the named theorems