**Metric Spaces Part 2**

**Exercises**

1. Show that Hoelder’s inequality implies Cauchy Schwartz (vector version)
2. Show that Minkowski’s inequality (vector version) may fail for $p < 1$.It would be okay to come up with a concrete example.
3. Prove that the function $d(m,n) = |1/m – 1/n|$ is a metric on the natural numbers
4. Let $x=\left\{\frac{5}{2^{n}}\right\}$ and $y=\left\{\frac{3}{2^{n}}\right\}$. Find $d(x,y)$ in $l^{2}$. Note: the answer is a single number
5. Define the vector spaces $l^{0}$, $l^{1}$, and $l^{p}$ for $p >1$. Give it your best shot (and model your definitions after $l^{2}$ and $R\_{0}^{n}, R\_{1}^{n}$, and $R\_{p}^{n}$). You do not need to prove anything, just say what you think a good definition might be.
6. For what vector spaces does the abstract definition of continuity reduce to our familiar $ϵ-δ$ one defined in calculus
7. If we take $f: C^{1}\left[0, 1\right]\rightarrow R\_{1}^{1}$ by defining (again): $f(x)=x(\frac{1}{2})$, is that operator $f$ continuous? How about $f(x)=x(\frac{1}{2})$ where $f:C^{2}\left[0,1\right]\rightarrow R\_{1}^{1}$
8. Let $m$ the space of all bounded sequences. Add a metric by defining $ρ\left(x,y\right)=sup⁡(\left|x\_{j}-y\_{j}\right|\}$. Then $m$ is a metric space. Define $L: m\rightarrow R\_{1}^{1}$ by setting $L(x)=sup⁡\{\left|x\_{j}\right|, j=1, 2, …\}$. Is $L$ continuous?
9. Show that the real line $(-\infty , \infty )$ and the interval $(-1, 1)$ are homeomorphic.
10. Is the map from example 2.4 1-1? How about onto? Is it a homeomorphism? How about the map from example 2.3? And how about the one from example 2.2?
11. Prove proposition 2.6: Any function that maps a discrete metric space to any metric space is continuous.
12. Find the (open and closed) unit ball for a discrete metric space X. Recall that the open unit ball is $\{x\in X: d(x,0)<1\}$ and the closed unit ball is $\{x\in X: d\left(x,0\right)\leq 1\}$
13. Let $S^{1}$ denote the unit circle in the plane: $S^{1}=\{\left(x, y\right)\in R^{2} :x^{2}+y^{2}=1\}$. Define a map $F : [0, 2π) \rightarrow S^{1}$ by $F(t) = (cos⁡(t),sin⁡(t))$. Prove that $F$ is continuous and bijective, but is not a homeomorphism.
14. Let $D\_{1}$ and $D\_{2}$ be the following disks in the plane (i.e. in $R^{2}$ with the usual Euclidian metric):

$$D\_{1}=\left\{\left(x, y\right): x^{2}+y^{2}<1\right\} and D\_{2}=\left\{\left(x, y\right): x^{2}+y^{2}<4\right\}$$

Show that $D\_{1}$ and $D\_{2}$ are homeomorphic.

1. Show that a bijective isometry between metric spaces is also a homeomorphism
2. Show that $C[0,1]$ and $C[1,2]$are isometric.