**Metric Spaces Part 2**

**Exercises**

1. Show that Hoelder’s inequality implies Cauchy Schwartz (vector version)
2. Show that Minkowski’s inequality (vector version) may fail for .It would be okay to come up with a concrete example.
3. Prove that the function is a metric on the natural numbers
4. Let and . Find in . Note: the answer is a single number
5. Define the vector spaces , , and for . Give it your best shot (and model your definitions after and , and ). You do not need to prove anything, just say what you think a good definition might be.
6. For what vector spaces does the abstract definition of continuity reduce to our familiar one defined in calculus
7. If we take by defining (again): , is that operator continuous? How about where
8. Let the space of all bounded sequences. Add a metric by defining . Then is a metric space. Define by setting . Is continuous?
9. Show that the real line and the interval are homeomorphic.
10. Is the map from example 2.4 1-1? How about onto? Is it a homeomorphism? How about the map from example 2.3? And how about the one from example 2.2?
11. Prove proposition 2.6: Any function that maps a discrete metric space to any metric space is continuous.
12. Find the (open and closed) unit ball for a discrete metric space X. Recall that the open unit ball is and the closed unit ball is
13. Let denote the unit circle in the plane: . Define a map by . Prove that is continuous and bijective, but is not a homeomorphism.
14. Let and be the following disks in the plane (i.e. in with the usual Euclidian metric):

Show that and are homeomorphic.

1. Show that a bijective isometry between metric spaces is also a homeomorphism
2. Show that and are isometric.