**Quiz #4: Lebesgue Outer Measure and Open Covers**

**Explain and/or Define:**

Riemann Integral

Difference between , , and (net) area under a curve

Lebesgue Outer Measure

Additive and subadditive set function

Countably additive and subadditive set function

Compact set (in an abstract topological space)

Heine Borel theorem (generally accepted version)

**Exercises:**

*All of the following questions are fair game for the quiz. However, you might want to especially focus on the highlighted one*

* + 1. Prove statement 1 on oddities of the Riemann integral, i.e. that value of an integral does not change if you change the value of the function at a single point.
    2. Identify the function mentioned in statement 2
    3. Provide the details for statement 6 on the oddities of the Riemann integral
    4. Prove statement 7 on the oddities of the Riemann integral
    5. If and are two sets with , then
    6. Find , i.e. the outer measure of a single point
    7. Find , i.e. the outer measure of the open interval (a, b).*Hint: use the fact that*
    8. Show that *Hint: use the fact that outer measure is monotone*
    9. Find the outer measure of an infinite interval
    10. What is the point of defining the Lebesgue outer measure. Why don’t we just use the length of a set instead of defining this strange and complicated outer measure. In other words, what is the advantage of outer measure over, say, length?
    11. Define the length of an interval from a to b as b – a. Is length subadditive or additive (a function is additive if for A, B disjoint, and subadditive if for any A, B).
    12. Define a set function (cm stands for “counting measure”) to be equal to the number of elements in , if is finite, or infinity. What is the domain and range of ? Is subadditive or additive? How about countably subadditive or additive?
    13. If is a compact set in , then contains its inf and sup
    14. Find an open cover of the interval (a, b) that can be reduced to a finite subcover. Does this create a problem with the Heine Borel theorem, since (a, b) is not closed?
    15. Find an open cover of the interval (a, b) that cannot be reduced to a finite subcover.
    16. Consider the interval and the collection for all and for all . Show that the cover the set S but you cannot reduce it to any finite subcover.