**Riemann Integration vs Lebesgue Integration**

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| **Riemann Integration**  We defined the Riemann integral of a bounded function defined on [a,b] roughly as follows:   * subdivide the ***domain*** of the function (usually a closed, bounded interval) into finitely many subintervals (the partition)      * construct *step functions* that has a constant value on each of the subintervals of the partition (the Upper or Lower sum) that are bigger or smaller than the function over each subinterval of the partition. The particular step functions to use are the upper and lower sums:   where  where   * If the upper and lower sum get closer and closer to each other as you increase the partition size in the domain, the function is Riemann integrable.   **Example**: Show that the function is Riemann integrable over by   1. partition the *domain* into 5 subintervals of length 0.4 2. drawing and 3. find | **Lebesgue Integration**  We defined the Lebesgue integral of a bounded function defined on a set E with finite measure roughly as follows:   * Subdivide the ***range*** of the function into finitely many pieces  * Construct *simple* *functions* by defining the sets (assuming, for example, ):   and set   * If the larger and smaller integrals of these simple functions sum get closer and closer to each other as you increase the points in the range, the function is Lebesgue integrable   **Example:** Show that the function is Lebesgue integrable over by   1. partition the *range* into 5 subintervals of width 1/5 2. draw the sets 3. write down and 4. find   How would you define the sets if . How about if you knew that was bounded by, say, |

**Questions:**

* 1. Why is it important for Riemann integration that the function f(x) is bounded?
  2. Prove that is Riemann integrable on the interval [0, 1]
  3. Modify the above proof to show that every continuous function on a closed, bounded interval [a, b] is Riemann integrable.
  4. Find
  5. Why is it important for Lebesgue integration that the function f(x) is bounded?
  6. Prove that is Lebesgue integrable on the interval [0, 1]
  7. Modify the above proof to show that every continuous function on a set with finite measure is Lebesgue integrable.
  8. Find

3. Which integration is more flexible, and why, conceptually? In other words, which statement is true and why:

1. If a bounded function f is L-integrable, then f is R-integrable
2. If a bounded function f is R-integrable, then f is L-integrable