**Lebesgue Integral of Bounded Functions**

***Exercises***

* 1. Are simple functions uniquely determined? In other words, if  and  are two simple functions with , do they have to have the same representation?
  2. Is it true that if  and  are two (measurable) sets then the characteristic function of the union of  and  is the sum of the characteristic functions of  and ? How about if  and  are two (measurable) sets then what is the characteristic function of the intersection of  and  equal to?
  3. Find the Lebesgue integral of the Dirichlet function restricted to [0, 1] and of the characteristic function of the Cantor middle-third set.
  4. Show that the function is *Riemann* integrable over [0,1] and find
  5. Show that the function is *Lebesgue* integrable over and find
  6. Is the function Lebesgue integrable over the set of rational numbers in ? If so find
  7. Identify the unnamed property referred to in the proof that is Lebesgue integrable.
  8. Repeat the proof that is measurable for over. Make sure to illustrate the proof by drawing the sets used in the proof.
  9. Prove that every bounded function defined on a measurable set with finite measure with the property that the sets

are measurable is Lebesgue integrable. Give an example of a class of functions that have this property and state it as a corollary of your theorem.

* 1. In the theorem of the previous exercise you assume that the function is bounded. Where do you need that fact in the proof?
  2. Is the converse of the above theorem that every R-integrable function is also L-integrable true or false (prove or provide counterexample.
  3. Prove that if a sequence of integrable functions converges uniformly to an integrable function on a set with finite measure then
  4. Is the Lebesgue bounded convergence theorem true for R-integrable functions?

PROJECT 2: Find a bounded function that is not Riemann integrable, and a bounded function that is not Lebesgue integrable.

PROJECT 3: Who is this guy Littlewood and what do his “three principles” say?