Analysis 2 – HW

1. Find the Fourier series of $f(x) = \begin{cases} 0, -\pi \le x \le 0 \\ \pi,\ \ 0 < x \le \pi \end{cases}$ Note that we found out in class already that $a_0 = \frac{\pi}{2}$, but you still need to find the rest. Make sure to use Maple to check if your series of, say, of up to 10 terms, will be close to the original function.

2. Find the Fourier series of $f(x) = \begin{cases} -x & , -\pi \le x \le 0 \\ x, & 0 < x \le \pi \end{cases}$. Make sure to use Maple to check your answer.

3. Repeat the previous question with $f(x) = \begin{cases} 0, -2 \le x \le 0 \\ x,\ \ 0 < x \le 2 \end{cases}$ Note that we already found the a_n's in class so you only need to work out the b_n. Also, not that the function is defined in $[-2,2]$, not in the "usual" interval. Make sure to use Maple to check your answer.

4. Suppose $\{f_n(x) = 0\}$ for all n and let f(x) be the Dirichlet function. Show that f_n converges to f a.e. How about f_n converging to f(x) for all x in [0, 1]. Does f_n mean-square converge to f, if we use the Riemann integral? How about if we use the Lebesgue integral?

5. Let $f_n(x) = e^{-n \cdot x2}$. Does f_n converge to f(x) = 0 a.e.? What about pointwise convergence to f(x) = 0? Is there a function f such that f_n converge pointwise to f for all x? Does the sequence of functions converge uniformly to the function $f(x) = \begin{cases} 1, & x = 0 \\ 0, & else \end{cases}$