Analysis 2 - HW

Note Title 3/30/2013

① In class we defined $C^2[a,b] = \{ f:[a,b] \to \mathbb{R},\ f \text{ continuous} \}$ with the metric $\rho(f,g) = \left(\int_a^b (f(t)-g(t))^2\, dt \right)^{1/2}$. Assume that the Schwarz inequality

$$\left(\int f(t)\, g(t)\, dt \right)^2 \leq \left(\int_a^b f^2(t)\,dt \right) \cdot \left(\int_a^b g^2(t)\,dt \right)$$

is true. Use it to prove the triangle inequality holds for the metric ρ.

② Prove the Schwarz inequality $\left(\int f(t)\, g(t)\, dt \right)^2 \leq \left(\int_a^b f^2(t)\,dt \right) \cdot \left(\int_a^b g^2(t)\,dt \right)$.

Hint: Start with

$$\int_a^b \int_a^b \left(f(t)g(s) - f(s)g(t) \right)^2 ds\,dt.$$

Expand, simplify, etc.

(3) For the set of continuous functions $C[a,b]$, we can define (at least) 3 metrics:

$$C^2[a,b] = (C[a,b], \rho) : \rho(f,g) = \left(\int_a^b |f(t) - g(t)|^2 \, dt \right)^{1/2}$$

$$C^1[a,b] = (C[a,b], \rho) : \rho(f,g) = \int_a^b |f(s) - g(s)| \, ds$$

$$C^0[a,b] = (C[a,b], \rho) : \rho(f,g) = \max_{a \le t \le b} \{ |f(t) - g(t)| \}$$

What does the unit sphere $\rho(f,0) \le 1$ look like?