Panel 1

Nested sphere Thm: If $S[x_j, r_j]$ closed nested spheres $\subset X$ with radius $r_j \to 0$, then

$$\bigcap_{j=1}^{n} S[x_j, r_j] \neq \emptyset \text{ iff } X \text{ is complete metric space.}$$

Last semester: intersection of compact, nested sets in $\mathbb{R}$ is not empty.

Dense sets: $\bar{A} = X$ ($\mathbb{Q}$ in $\mathbb{R}$)

Nowhere dense sets: $\overline{\mathrm{compl}(\bar{A})}$ X ($\mathbb{N}$ in $\mathbb{R}$)

$\Leftrightarrow$ for every sphere S there is sphere $S' \subset S$ s.t. $S' \cap A = \emptyset$

1

Panel 2

Sets of 1^{st} category: can be written as countable union of nowhere dense sets.
"meager"

Sets of 2^{nd} category: not cat. 1

Baire's Category Thm: Every complete metric space is of 2^{nd} category!

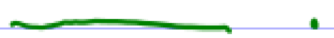
Consequence I: If X (complete metric space) has no isolated points $\Rightarrow$ uncountable!

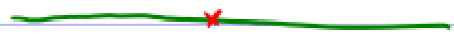
$\{x\}$ is nowhere dense!

2

Panel 3

$\{x\}$ is not isolated $\Rightarrow$ $\overline{\text{compl}\{x\}} = X$

 X $\Rightarrow$ $\{x\}$ is not nowhere dense

 X $\Rightarrow$ $\{x\}$ is nowhere dense

X was countable union of $\{x\}$

$\Rightarrow$ X is 1^{st} category!

$[0,1)$ is uncountable

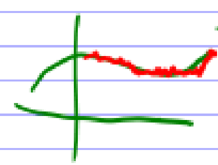
$[0,1] \cup \{2\}$

3

Panel 4

If P inf. diff'ble function, for each x $\exists$ n s.t. $p^{(n)}(x) = 0$ $\Rightarrow$ P is a polynomial!

$C[0,1]$ set of nowhere diff'ble functions is dense in C^0



There exists a continuous $f:[0,1]\to\mathbb{R}$ that is not monotone on any interval of positive length!

There is a period cont. function whose Fourier Series diverges on an uncountable set.

There is a PDE with no solution!

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