Panel 1

Last Time

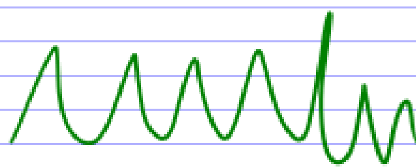
decimal $\leftrightarrow$ binary

characters $\leftrightarrow$ binary

images $\leftrightarrow$ binary

sound $\leftrightarrow$ binary

soundwave



$\rightarrow$ to sequence of #'s

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Panel 2

Task: Convert function to sequence of numbers!

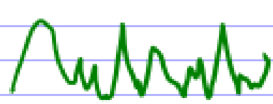
Idea I. $f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n$

store all a_i's : $a_0, a_1, a_2, \dots a_\infty$

Problem using $f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n$:

only very nice functions have series (real analytic)

sound waves tend to be – well – wavy



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Panel 3

Power Series: $\underbrace{f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n}$ $\Rightarrow a_n = \frac{f^{(n)}(c)}{n!}$.

Fourier Series:

$$f(x) \sim a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$$

Given that $f(x)$ has Fourier Series,

(1) Fourier coefficients a_n, b_n

(2) Convergence of Fourier series

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Panel 4

Some computations:

$$\int_{-\pi}^{\pi} \sin(nx)\,dx = -\frac{1}{n}\cos(nx)\Big|_{-\pi}^{\pi} = 0 \quad \forall n$$

$$\int_{-\pi}^{\pi} \cos(nx)\,dx = 0 \quad \forall n$$

$$\int_{-\pi}^{\pi} \sin(nx)\cos(mx)\,dx = 0 \quad \forall n,m$$

$$\int_{-\pi}^{\pi} \sin^2(nx) = \pi \qquad \int_{-\pi}^{\pi} \sin(nx)\cos(mx)\,dx = 0$$

$\forall n,m,\ n \neq m$.

$$\int_{-\pi}^{\pi} \cos^2(nx) = \pi$$

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Panel 5

If $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$ then:

int. all $\int_{-\pi}^{\pi} \sin(mx)\,dx$

$$\int_{-\pi}^{\pi} f(x)\sin(mx)\,dx = \int_{-\pi}^{\pi} \cancel{a_0 \sin(mx)}\,dx + \sum_{n=1}^{\infty}\left[\int_{-\pi}^{\pi} a_n \cancel{\cos(nx)\sin(mx)}\,dx + \int_{-\pi}^{\pi} b_n \sin(nx)\sin(mx)\,dx\right]$$

$$= \pi b_m$$

$$\Rightarrow b_m = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(mx)\,dx$$

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Panel 6

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx) \qquad \Big|\cdot\cos(mx),\ \int$$

$$\int_{-\pi}^{\pi}\cos(mx) f(x) = \int_{-\pi}^{\pi} a_0 \cancel{\cos(mx)} + \sum_{n=1}^{\infty}\int_{-\pi}^{\pi} a_n \cos(nx)\cos(mx) + \int_{-\pi}^{\pi} b_n \cancel{\sin(nx)\cos(mx)}$$

$$\int_{-\pi}^{\pi} f(x)\cos(mx)\,dx = \pi a_m$$

$$\Rightarrow a_m = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(mx)\,dx$$

$$\int_{-\pi}^{\pi} f(x) = \int_{-\pi}^{\pi} a_0 + \sum\int_{-\pi}^{\pi} \cancel{a_n\cos(nx)} + b_n\int_{-\pi}^{\pi}\cancel{\sin(nx)} \quad \Rightarrow a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

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Panel 7

Thm: If $f(x) = a_0 + \sum a_n \cos(nx) + b_n \sin(nx)$
then

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$$

Recall: If $f(x) = \sum a_n x^n \Rightarrow a_n = \frac{f^{(n)}(0)}{n!}$

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Panel 8

Ex: Say $f(x) = x$, $x \in [-\pi, \pi]$. Find its Fourier series, assuming it has one.

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \overset{\text{odd}}{x}\,dx = \frac{1}{2\pi}\,\frac{1}{2}x^2\Big|_{-\pi}^{\pi} = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \overbrace{x\cos(nx)}^{\text{odd}}\,dx = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x\sin(nx)\,dx = \frac{1}{\pi}\left(-\frac{1}{n}x\cos(nx)\Big|_{-\pi}^{\pi} + \frac{1}{n}\cancelto{0}{\int_{-\pi}^{\pi}\cos(nx)\,dx}\right) =$$

$u = x$ $\quad u' = 1$

$v' = \sin(nx)$ $\quad v = -\frac{1}{n}\cos(nx)$

$$= -2\frac{\pi}{n}\cos(n\pi) = -2\frac{\pi}{n}(-1)^n = 2\frac{\pi}{n}(-1)^{n+1}$$

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Panel 9

Ex. $f(x) = x, \quad x \in [-\pi, \pi]$

$a_j = 0 \;\; \forall j, \quad b_j = \frac{2}{j}(-1)^{j+1}$

$$x = \sum_{j=1}^{\infty} \frac{2}{j}(-1)^{j+1} \sin(jx) = 2\sin(x) - \frac{2}{2}\sin(2x) + \frac{2}{3}\sin(3x) - \ldots$$

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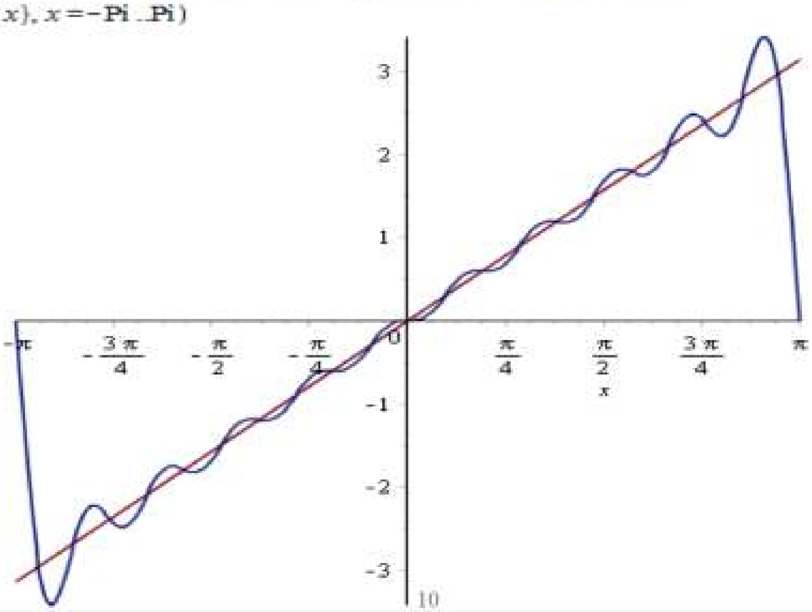
Panel 10

$sum\left(\frac{2 \cdot (-1)^{(n+1)}}{n} \cdot \sin(n \cdot x), n = 1..10\right)$

$$2\sin(x) - \sin(2x) + \frac{2}{3}\sin(3x) - \frac{1}{2}\sin(4x) + \frac{2}{5}\sin(5x) - \frac{1}{3}\sin(6x) + \frac{2}{7}\sin(7x) - \frac{1}{4}\sin(8x) + \frac{2}{9}\sin(9x) - \frac{1}{5}\sin(10x)$$

$plot(\{\%, x\}, x = -Pi..Pi)$



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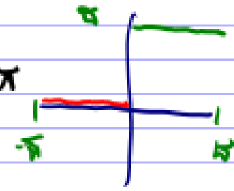
Panel 11

Ex: LA $f(x) = \begin{cases} 0 & , -\pi \leq x < 0 \\ \pi & , 0 \leq x < \pi \end{cases}$



Recall: $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$

$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$ $a_n =$ HW $b_n =$ HW

$= \frac{1}{2\pi}\int_0^{\pi} \pi\,dx = \frac{1}{2\pi}\pi^2 = \pi/2$

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Panel 12

$f(x) = \begin{cases} 0 & , -\pi \leq x < 0 \\ \pi & , 0 \leq x < \pi \end{cases}$

$a_0 = \pi/2$, $a_n = 0$, $b_n = \frac{1}{n}\left(1-(-1)^n\right)$

$\Rightarrow f(x) = ?$

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Panel 13

$f(x) := \frac{\text{Pi}}{2} + 2 \cdot sum\left(\frac{\sin((2n+1) \cdot x)}{2n+1}, n = 0..10 \right)$

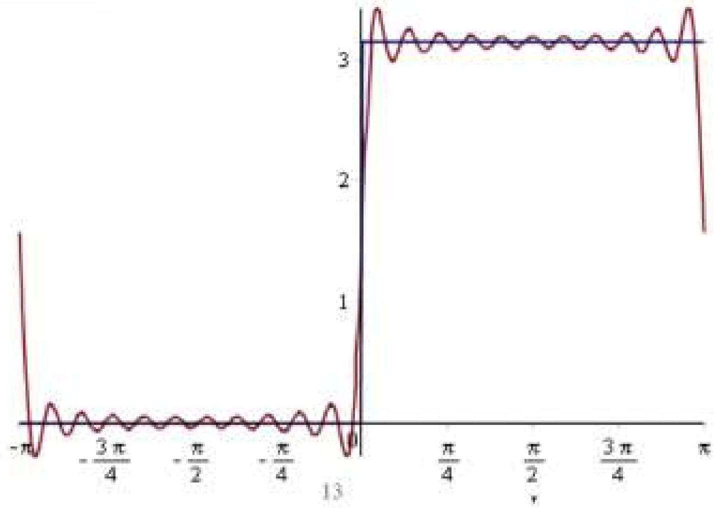
$$x \rightarrow \frac{1}{2}\pi + 2\left(\sum_{n=0}^{10} \frac{\sin((2n+1)x)}{2n+1} \right)$$

$g(x) := piecewise(x < 0, 0, \text{Pi})$

$$x \rightarrow piecewise(x < 0, 0, \pi)$$

$$1.512481142$$

$plot(\{f(x), g(x)\}, x = -\text{Pi}..\text{Pi})$



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Panel 14

So what!

Recall: $\sin(440x)$ is "A" sound

$$f(x) = a_0 + a_1\cos(x) + b_1\sin(x) + a_2\cos(2x) + b_2\sin(2x) + \ldots a_{440}\cos(440x) + b_{440}\sin(440x) + \ldots$$

If $b_{440} \neq 0$, soundwave includes "A" sound

→ Boost Base ⇒ increase a_j, b_j, for small j

→ low-pass filter ⇒ remove all a_j, b_j, j larger than 1000

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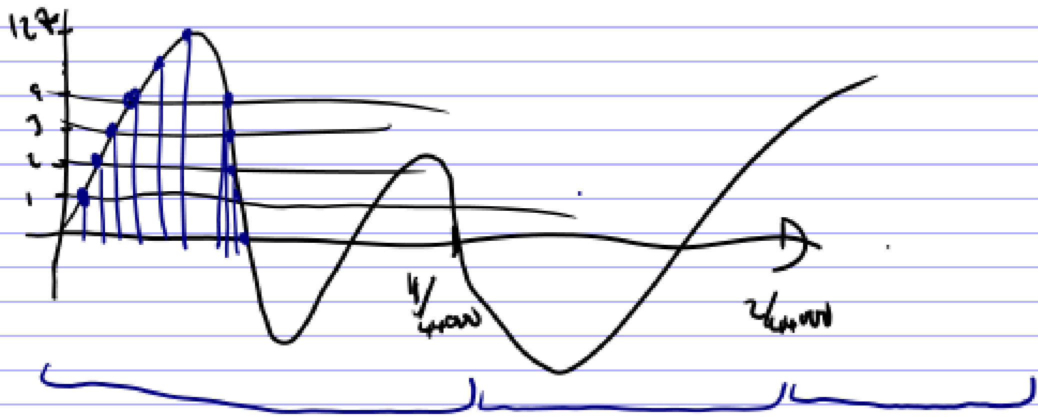
Panel 15

What does this have to do with a

44Khz, 16 bit Wav file

Sound wave: sample every 44 000 hz $= \frac{1}{44000}$ secs



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