Panel 1

Last Time:

$f^{-1}((a,b))$ is measurable if f is cont.

$\Rightarrow$ $f^{-1}((a,b])$, $f^{-1}([a,b))$, $f^{-1}([a,b])$ msble!

If f is L. integrable and $\mu(E) = 0$ then

$$\int_E f \, d\mu = 0$$

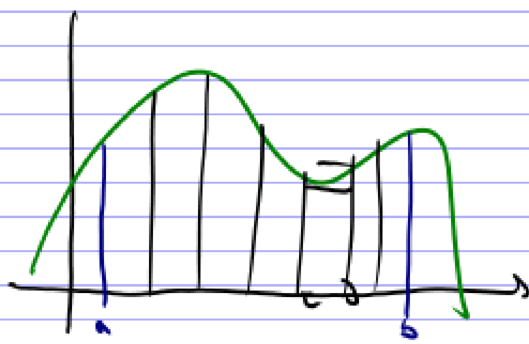
long proof!

$\int_{[0,1]} x \, d\mu$ is Leb. intble, and

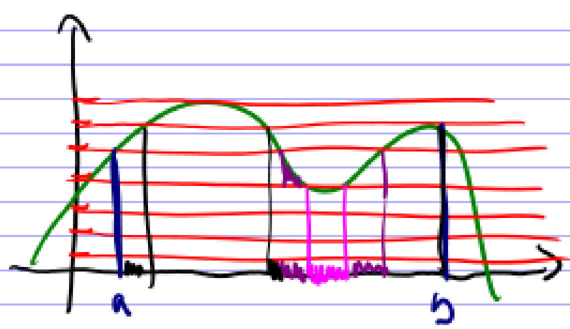
$= \frac{1}{2}$

1

Panel 2



Riemann: partition the interval $[a,b]$
$\Rightarrow$ step functions



Lebesgue!

2

Panel 3

Is *every* bounded function Lebesgue integrable?

Context

That would be too nice to be true - therefore it is not true! But it is difficult to find a bounded function that is not Lebesgue integrable (whereas it is easy to find a bounded function that is not *Riemann* integrable).

We have said before that we can prove that a bounded function with the property that the inverse image of a measurable set is measurable would be Lebesgue integrable. To find a bounded function that is not integrable we therefore need to find a function for which that property is not true.

If $C(x)$ is the Cantor function defined in chapter 6, then let $f(x) = C(x) + x$. It can be shown that f has bounded inverse function $g = f^{-1}$ and that there exists a measurable set A such that $g^{-1}(A)$ is not measurable. That function turns out to be a bounded function which is not Lebesgue integrable.

3

Panel 4

Thm: If f is a bounded function on $[a,b]$, then f is R-intble then f is L-intble and

$$\int_a^b f(x)\,dx = \int_{[a,b]} f(x)\,d\mu$$

$$I^*(f)_R = \inf\{ \sum c_i \Delta x_i ,\ c_i \geq f \}$$

$$I_*(f)_R = \sup\{ \sum d_i \Delta x_i ,\ d_i \leq f \}$$

Every step function is also simple!!! Leb. intble

$$\underbrace{I_*(f)_R \leq \underbrace{I_*(f)_L \leq \cancel{\int f\,dx} \leq I^*(f)_L}_{\text{equal}} \leq I^*(f)_R}_{\text{equal}}$$

4

Panel 5

? Find the Lebesgue integral of $f(x) = x \cos(x)$ over the interval $[-1, 1]$.

? Show that the converse of the above theorem is false, i.e. not every bounded Lebesgue integrable function is Riemann integrable.

? If possible, find the Riemann and Lebesgue integrals of the constant function $f(x) = 1$ over the Cantor middle-third set.

? ~~Show that the restriction of a bounded continuous function to a measurable set is Lebesgue integrable.~~

HW

Panel 6

At this point we could stop: we have extended the concept of integration to (bounded) functions defined on general sets (measurable sets with finite measure) without using partitions (subintervals). The new concept, the Lebesgue integral, agrees with the old one, Riemann integral, when both apply, and it removes some of the oddities mentioned before.

Should define general Lebesgue int.
for f not necessarily bdd!!

Panel 7

And Now for Something completely different!

http://www.youtube.com/watch?v=K2P86C-1x3o

http://www.youtube.com/watch?v=al_iBlpTn80&NR=1&feature=fvwp

http://en.wikipedia.org/wiki/And_Now_for_So…

Panel 8

How to "save" or "record" stuff:

Memorization

||||||||||||

enhance: ~~||||~~ ~~||||~~ ||

enhance $\overline{V}\ \overline{V}\ \overline{II}\ \overline{X}\ L$

enhance 1230 need experts

enhance 12.345

enhance $\frac{1}{\sqrt{2\sigma}} \int e^{-x^2} dx$

Panel 9

Other symbols: a b c d ...

form words + meaning

elementary school

lots of training to

encode and decode

writing reading

9

Panel 10

Painting:

encoding with paint + brush

Experts!

decoding is simple: I can see

no training necessary,

not perfect!

10

Panel 11

Roy Lichten-stein

11

Panel 12

Audio / Music

notes on paper to encode
instruments + players to decode

record it analog via record player
reply it.

12

Panel 13

Panel 14

Photography

chemically treated paper, different colors show up depending on exposure to light

specialist : lab for prints!

14

Panel 15

Storage Today

all ~~is~~ digital: all things are 0's, 1's!

How???

15