Panel 1

Complex Homework #20

(1) Without actually finding the series, find the radius of convergence for each Taylor series:

a) $\frac{1}{3-z} = \sum_{n=0}^{\infty} a_n z^n$

b) $\frac{1}{(z+2i)(z-4)} = \sum_{n=0}^{\infty} a_n (z-1)^n$

c) $\frac{1}{e^z+1} = \sum_{n=0}^{\infty} a_n z^n$

d) $\text{Log}(z-i) = \sum_{n=0}^{\infty} a_n z^n$

(2) Find Taylor series at the given center z_0 for:

a) $f(z) = \sinh(z)$, $z_0 = 0$

b) $g(z) = z^5 e^{2z}$, $z_0 = 0$

c) $h(z) = \frac{z^3}{1+z^3}$, $z_0 = 0$

d) $k(z) = \frac{1}{z-4}$, $z_0 = 2$

Panel 2

(3) Find the Laurent series centered at $z_0 = 0$, and list specifically the value of the coefficient $a_{-1} = \_\_\_$

a) $f(z) = z^4 \sin(1/z)$

b) $g(z) = \frac{1}{z^4} \sin(z)$

(4) The function $f(z) = \frac{1}{3-z}$ has two series expansions, one for $|z| < 3$ and another for $|z| > 3$. Find them.

(5) $f(z) = \frac{-2}{(z-1)(z-3)}$ has 3 series expansions. Find all three series and their domain of convergence (centered at $z_0 = 0$)

(6) How many series expansions centered at $z_0 = 0$ does $f(z) = \frac{1}{e^z - 1}$ have, and where do they converge?