Panel 1

Complex Analysis HW

(1) Recall Liouville's Thm and its proof. Use a similar idea to show that if f is entire and $|f'(z)| \leq M|z| \quad \forall z$ for some M, then $f(z) = az$

(2) Suppose f is entire, $|f'(z)| \leq 1 \quad \forall z$, and $f(0) = f(1) = 0$. Show that $f(z) \equiv 0$
(Hint: apply Liouville's thm to f' if possible)

(3) Find $\int_\gamma \frac{\sin(z)}{z^2}\,dz$, γ a pos. oriented curve around zero.

Panel 2

(4) Let C be the ellipse $9x^2 + 4y^2 = 36$, counterclockwise oriented. Let

$$g(z) = \int_C \frac{s^2 + s + 1}{s - z}\,ds$$

Find a) $g(i)$ and
b) $g(4i)$

(5) Find $\int_{|z|=4} \frac{\cos(z)}{z^2 - 6z + 5}\,dz$