

Panel 1

Complex HW

① We defined $e^z = e^x e^{iy} = e^x (\cos(y) + i \sin(y))$

a) Solve $e^z = -3$

b) Show that e^z is entire while $e^{\bar{z}}$ is nowhere differentiable

② If $z = re^{it}$ we defined $\log(z) = \ln(r) + i(t + 2k\pi)$ and $\text{Log}(z) = \ln(r) + it$.

a) Find $\log(i)$, $\text{Log}(i)$, $\log(-3)$, $\text{Log}(-3)$, $\text{Log}(1+i)$

b) Show that $\log(z^i) \neq i \log(z)$ but $\text{Log}((1+i)^i) = 2 \cdot \text{Log}(1+i)$

Panel 2

③ We defined $\cos(z) = \frac{e^{iz} + e^{-iz}}{2}$, $\sinh(z) = \frac{e^z - e^{-z}}{2i}$

a) Find $\cos(i)$ and $\sinh(\pi i)$

b) Solve $\cos(z) = i$ and $\sinh z(\pi) = 3$

c) Show that $\sinh^2(z) + \cos^2(z) = 1$

④ Find the following values:

a) i^i

b) 3^i

c) i^{i^i} , i.e. $(i^i)^i$, $i^{(i^i)}$

⑤ Find $\lim_{t \rightarrow \infty} e^{it}$ and $\lim_{t \rightarrow \infty} \cos(it)$