Panel 1

State Euler's Theorem regarding e^{it}. $e^{it} = \cos(t) + i\sin(t)$

State the definition of za complex function is **C**-differentiable.

$\lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$ exists

What does it mean if a function $f: C \to C, f(z) = u(x,y) + iv(x,y)$ satisfies the **Cauchy Riemann** equations. $u_x = v_y, \quad u_y = -v_x \qquad f' = u_x + iv_x$

What is the definition of a function being analytic at a point $z_0 = x_0 + iy_0$?

What is the harmonic conjugate to a harmonic function u(x, y)

There is v s.t. $u + iv$ is analytic.

Panel 2

Chapter 3: exp and Friends

Def: $e^z = e^{x+iy} = e^x e^{iy} = e^x(\cos(y) + i\sin(y))$

Properties:

$e^z \neq 0$

$|e^z| = e^x |e^{it}| = e^x \cdot 1 = e^x$

e^z is periodic with period $2\pi i$ $\quad e^z = e^x e^{iy} = e^x e^{i(y+2k\pi)}$

i.e. $e^z = e^{z + 2k\pi i}$

$\frac{d}{dz} e^z = u_x + iv_x = e^x\cos(y) + i\,e^x\sin(y) = e^x e^{iy} = e^z$

Panel 3

Ex: Solve $e^{z} = 1+i = \sqrt{2}e^{i\pi/4}$

$e^{x}e^{iy}$

$\Rightarrow e^{x} = \sqrt{2}, \quad y = \frac{\pi}{4} + 2k\pi$

$\Rightarrow x = \frac{1}{2}\ln(2)$

inf. many solutions

Solve $e^{z} = -1 = e^{i\pi} \quad \Rightarrow z = i\pi + i2k\pi$

So e^{z} can be negative but only for imag. values of z!

Panel 4

Def: If $z = re^{i\theta}$, define $\log(z) = \ln(r) + i(\theta + 2k\pi)$

because $e^{\log(z)} = e^{\ln(r)}e^{i(\theta+2k\pi)} =$

$= r\,e^{i\theta}\cancel{e^{i2k\pi}} = re^{i\theta} = z$ ✓

$\log(z)$ is not a function, because it has inf. many values for any z.

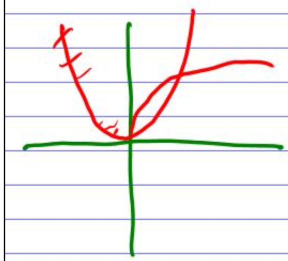
Note: $\log(e^{z}) = \log(e^{x}e^{iy}) = \ln(e^{x}) + i(y + 2k\pi) =$
$x + iy + 2k\pi i \neq z$

Panel 5

$\sqrt{x}$ and x^2: $(\sqrt{x})^2 = x$

$\sqrt{x^2} = |x|$

$\sqrt{x}$ and x^2 are NOT inverse. But if I restrict x^2 to only $x \geq 0$, then it does have inverse.



Panel 6

Ex: $Log(1) = log(e^{i0}) = ln(1) + i(0 + 2k\pi) = 2k\pi i$

Def: The principle value of the logarithm is:

$$Log(z) = ln(r) + i\theta, \text{ where } z = re^{i\theta}.$$

(k=0 in prev. def)

Properties:

$Log(z)$ is a function

$e^{Log(z)} = Log(e^z) = z$ ✓

$\frac{d}{dz} Log(z) = \frac{1}{z}$

Panel 7

Ex: $\ln(x)$ is really defined for all $x \neq 0$

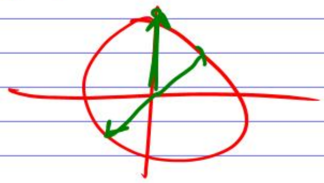
$$\log(-1) = \log(re^{it}) = \log(1e^{i\pi}) = \ln(1) + i(\pi + 2\pi k)$$

$$\mathrm{Log}(-1) = i\pi$$

Other interesting facts:

$$i^2 = -1$$

$$(i)^{1/2} = e^{i\pi/4} \rightarrow (e^{i\pi/4})^2 = e^{i\frac{\pi}{2}} = i \checkmark$$



$$(e^{i\frac{5\pi}{4}})^2 = e^{i\frac{5\pi}{2}} = e^{i\frac{\pi}{2} + 2\pi i} = i$$

Panel 8

$$2^i = e^{i\ln(2)} = e^{\ln(2^i)}$$

$$= \cos(\ln(2)) + i\sin(\ln(2))$$

$$i^i = e^{i\ln(i)} = e^i$$

$$\ln(i) = \ln(1e^{i\frac{\pi}{2}})$$

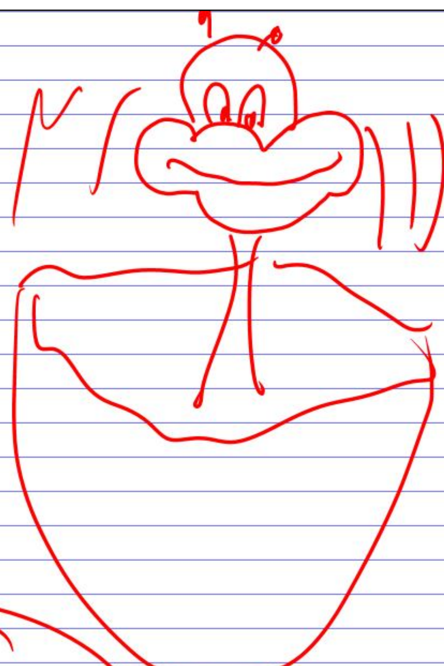
$$= \ln(1) + i\frac{\pi}{2}$$

$$i^i = e^{i\ln(i)} = e^{i(i\frac{\pi}{2})} = e^{-\frac{\pi}{2}}$$

i^i is real ?!!!

i^{i^i}

$2^{i^i} \rightarrow ?$



Panel 9

Derivative of log

Panel 10

Trig Functions:

$\sin(z) = \frac{e^{iz} - e^{iz}}{2i}$ $\quad \frac{d}{dz}\sin(z) = \cos(z)$

$\cos(z) = \frac{e^{iz} + e^{-iz}}{2}$ $\Rightarrow \frac{d}{dz}\cos(z) = \frac{ie^{iz} + (-i)e^{-iz}}{2} = \frac{i(e^{iz} - e^{-iz})}{2}$

$1/i = -i$ $\quad = \frac{-(e^{iz} - e^{-iz})}{2i} = -\sin(z)$

Know: $e^{it} = \cos(t) + i\sin(t)$

$e^{-it} = \cos(-t) + i\sin(-t) = \cos(t) - i\sin(t)$

$\Rightarrow e^{it} + e^{-it} = 2\cos(t) \Rightarrow \cos(t) = \frac{e^{it} + e^{-it}}{2}$

$\sinh(x) = \frac{e^{x} - e^{-x}}{2}$

Panel 11

$|\sin(x)| \le 1 \quad |\cos(x)| \le 1$

$$\lim_{t\to\infty} \cos(it) = \lim_{t\to\infty} \frac{e^{i(it)} + e^{-i(it)}}{2} =$$

$$= \lim_{t\to\infty} \frac{e^{-t} + e^{t}}{2} = \infty$$

$\cos(z)$ is unbounded!!!!

Solve $\cos(z) = 2$? Ponder this!

Maybe approximately!

Panel 12

About e^z, $\cos(z)$, $\sin(z)$

e^z is periodic with period $2\pi i$

$\sin(z)$, $\cos(z)$ are unbounded along imag axis!

There is no sin, cos!

There is only e^z