

Panel 1

## Complex HW #2

① Show that the following functions are not  $\mathbb{C}$ -diffble:

a)  $f(z) = 2x + iy^2$

b)  $f(z) = z - \bar{z}$

c)  $f(z) = e^x e^{-iy}$

Hint: check CR equations

② Use the CR equations to show that  $f'(z)$  exists for

$f(z) = z^3$  and verify that  $f'(z) = 3z^2$

③ Let  $f(z) = x^3 + i(1-y)^3$ . Show that  $f$  is  $\mathbb{C}$ -diffble

only for  $z = i$  and find  $f'(z)$  for  $z = i$

④ Suppose  $f(z)$  is  $\mathbb{C}$ -diffble in a domain  $D$  and  $f(z)$  is real-valued. Prove that  $f(z)$  must be constant.

(Real-valued means that  $f(z) \in \mathbb{R} \forall z \in D$ ) Hint: CR