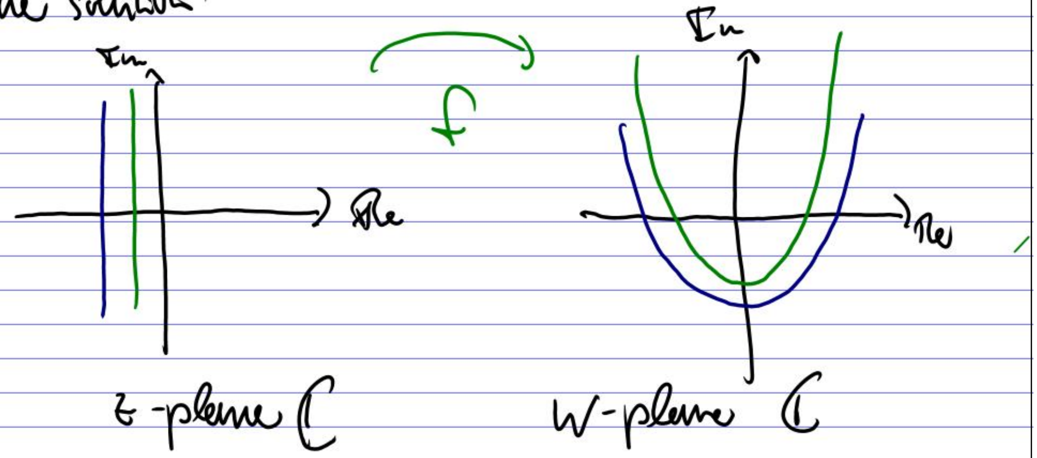
Panel 1

Graphs of Complex Functions

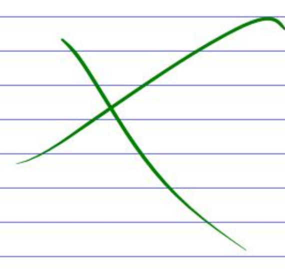
Problem : $f: \mathbb{C} \to \mathbb{C}$
$f: \mathbb{R}^2 \to \mathbb{R}^2$ graph in 4D !!!

One solution:



Panel 2

Graphing Complex Functions



Panel 3

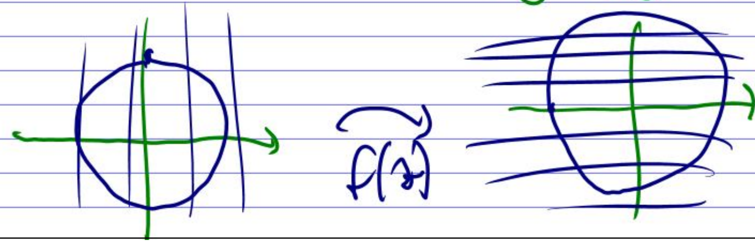
Ex: Describe the mapping properties of mult. by i, i.e.

$$f(z) = iz$$

$z = x + iy \qquad f(z) = iz = ix - y$?

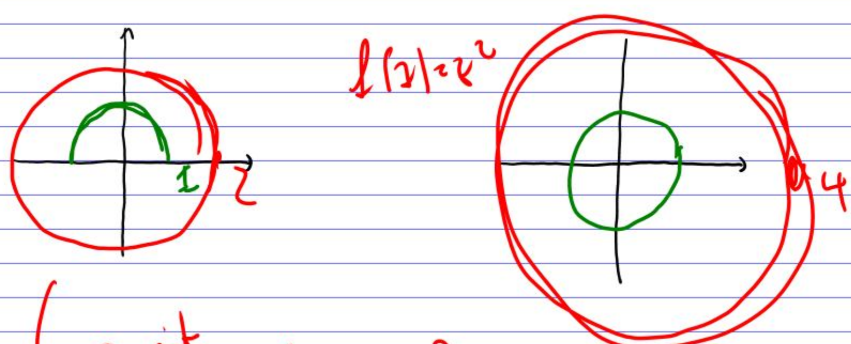
$z = re^{i\theta} \qquad f(z) = ire^{i\theta} =$

$$= e^{i\frac{\pi}{2}} re^{i\theta} = re^{i(\theta + \frac{\pi}{2})}$$



Panel 4

Ex: What does $f(z) = z^2$ do to circles and radii?

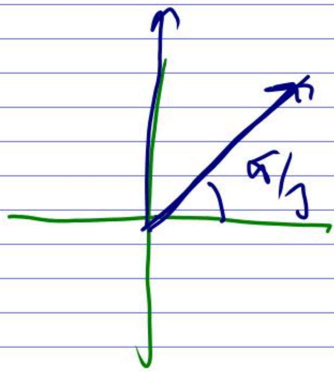


$2e^{it} \Rightarrow t \in [0, 2\pi)$

$f(2e^{it}) = (2e^{it})^2 = 4e^{2it}, \quad t \in [0, 2\pi)$

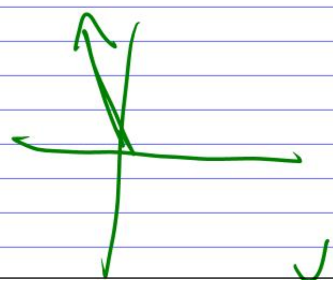
Panel 5

$f(z) = z^2$ maps radii



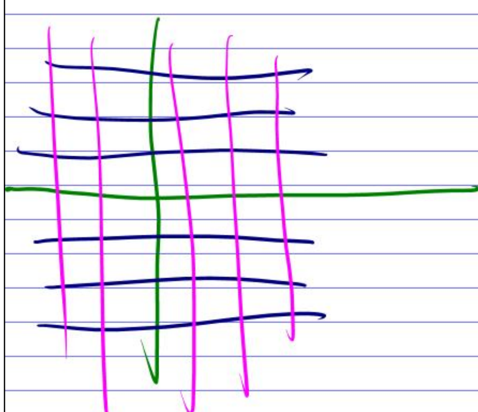
$r e^{i\pi/3}$, $r \in [0, \infty)$

$f(r e^{i\pi/3}) = r^2 e^{i 2\pi/3}$



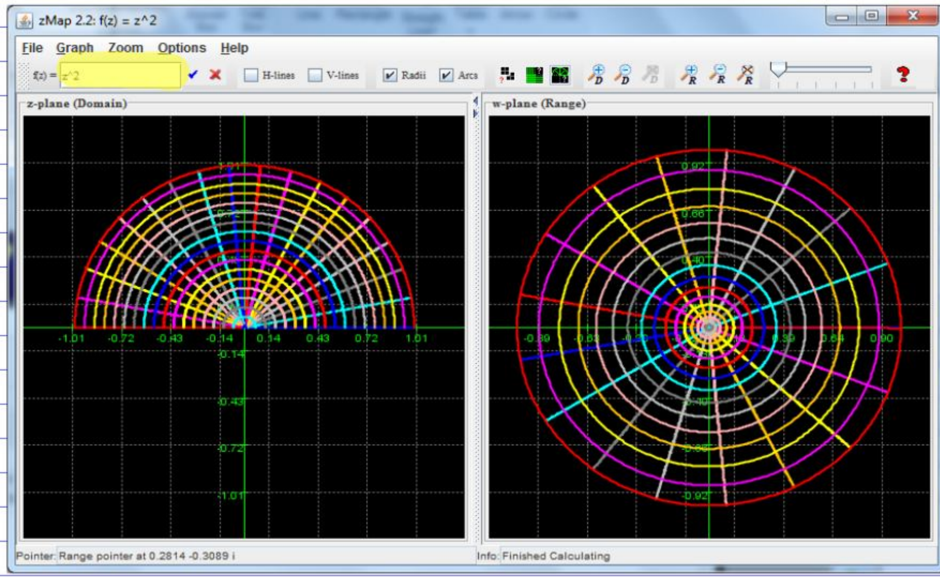
Panel 6

$f(z) = z^2$



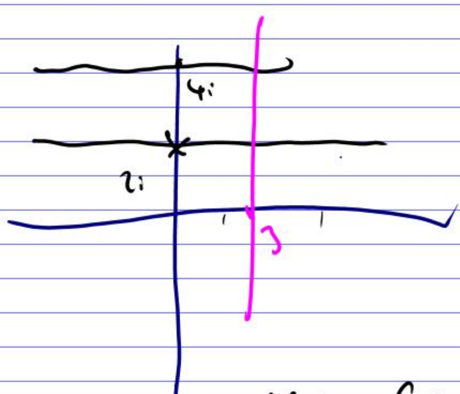
Panel 7

zMap: Program to visualize complex functions



Panel 8

Horizontal line:



$\left.\begin{matrix} t+2i \\ t+4i \end{matrix}\right\}$ horiz. lines

$3+ti \}$ vert. line

$$f(z) = (t+2i)^2 = \underbrace{t^2-4}_{x} + \underbrace{4ti}_{y}$$

$y = 4t \Rightarrow t = \frac{y}{4}$

$x = t^2 - 4 \qquad x = \frac{y^2}{16} - 4$

Panel 9

$f(z) = e^z$ maps horiz. lines

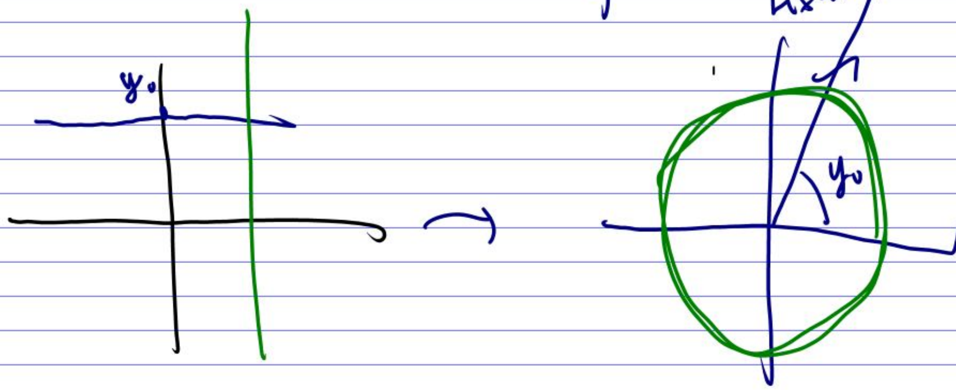
$\ell(t) = t + i y_0 = z$

$e^z = e^{t+iy_0} = e^t \cdot e^{iy_0}$

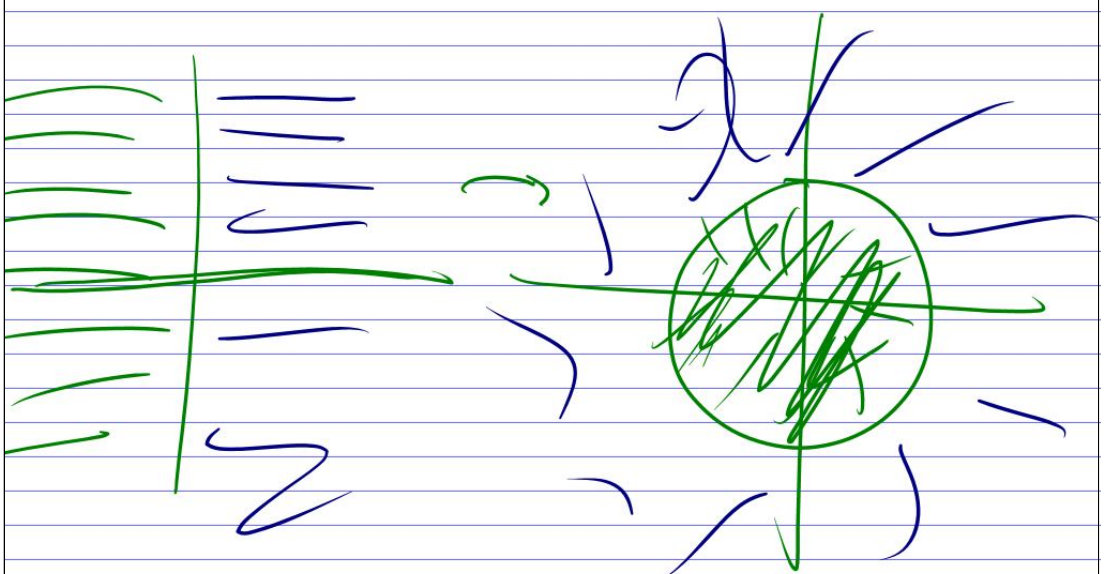
$\ell(t) = x_0 + ti = z$

$e^z = e^{x_0} e^{ti}$

fixed



Panel 10

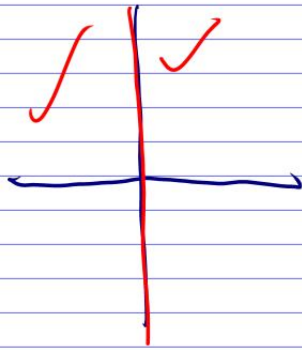


Panel 11

Domain: $f(z)=\frac{z}{z+\bar{z}}$ $\forall z$ except

$z+\bar{z}=0$

$2x=0$



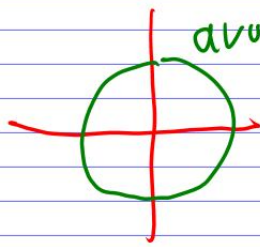
$f(z)=\frac{1}{1-z\bar{z}}$ $\forall z$ except

$1-z\bar{z}=0$

avoid!

$1=z\bar{z}=|z|^2$

$=x^2+y^2$



Panel 12

$f(z)=z+1/z$

$f(re^{it})=re^{it}+\frac{1}{r}e^{-it}$

$=r(\cos(t)+i\sin(t))+\frac{1}{r}(\cos(-t)+i\sin(-t))$

$=r\cos(t)+ri\sin(t)+\frac{1}{r}\cos(t)-\frac{1}{r}\sin(t)$

$=\underbrace{\cos(t)\left(r+\frac{1}{r}\right)}+i\sin(t)(r-1/r)$

$u(r,t)+iv(r,t)$