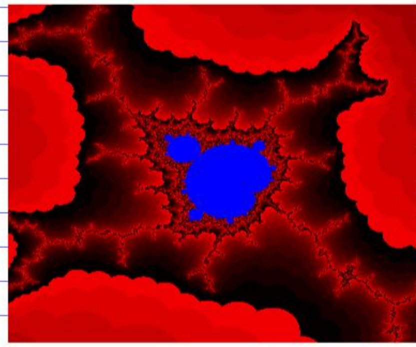
Panel 1

Welcome to Math 4512

Complex Analysis



(where $z^2 + 1 = 0$ does have solutions
and numbers are no longer sorted)

Panel 2

Math 4512 : Complex Analysis

Instructor: Bert Wachsmuth
wachsmut@shu.edu ~~(x5164)~~
Office : MWF 11-12
by appointment

Grading: 2 exams (45%)
Homework (45%)
Participation (10%)
At least one presentation

Panel 3

Other Stuff: http://pirate.shu.edu/~wachsmut
Dyknow Server
dyknows://secure.dyknow.com/shu.edu
User: 8-letter pass: 8-letters again

Material covered:
Complex Numbers + Algebra
Complex Functions: limits, cont., deriv., integration
Series (!)
Residues + Applications.

Panel 4

Definition of Complex Numbers:

Pairs $z = (x, y)$ where x, y are real, and the sum and product defined as:

$z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$

$z_1 + z_2 = (x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$

$z_1 \cdot z_2 = (x_1, y_1) \cdot (x_2, y_2) = (x_1 x_2 - y_1 y_2, x_1 y_2 + x_2 y_1)$

$z = (x, y)$; $\mathrm{Re}(z) = x$, $\mathrm{Im}(z) = y$
(imaginary

Note: $0 = (0, 0)$

Panel 5

$z_1 + z_2 = (x_1, y_1) + (x_2, y_2) =$

$z_1 \cdot z_2 = (x_1, y_1) \cdot (x_2, y_2) = (x_1 x_2 - y_1 y_2, x_1 y_2 + x_2 y_1)$

Why not: $(x_1, y_1) \cdot (x_2, y_2) \overset{*}{=} (x_1 x_2, y_1 y_2)$

Want: $z_1 \cdot z_2 = 0 \Rightarrow z_1 = 0 = (0,0)$ or $z_2 = 0 = (0,0)$

If mult. is defined by $*$ then

$(1,0) \cdot (0,5) = (\cancel{0,0})$ not good

Ex: $(1,0) \cdot (0,5) = (1 \cdot 0 - 0 \cdot 5, 1 \cdot 5 + 0 \cdot 0) = (0,5)$

Panel 6

$z_1 + z_2 = (x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$

$z_1 \cdot z_2 = (x_1, y_1) \cdot (x_2, y_2) = (x_1 x_2 - y_1 y_2, x_1 y_2 + x_2 y_1)$

What if $z_1 = (x_1, 0)$ and $z_2 = (x_2, 0)$?

$\Rightarrow z_1 + z_2 = (x_1 + x_2, 0)$

and $z_1 \cdot z_2 = (x_1 x_2, 0)$

Thus: $\mathbb{R} \subset \mathbb{C}$, $\mathbb{C} = \{$set of complex numbers$\}$

Same idea: $\mathbb{N} \subset \mathbb{Q}$, $5 = \frac{5}{1}$

Panel 7

What's the Deal (I mean, really)?

Take $z_1 = (1,2)$ and $z_2 = (3,4)$, find $z_1 \cdot z_2$.

$(1,2)\cdot(3,4) = (1\cdot 3 - 2\cdot 4,\ 1\cdot 4 + 2\cdot 3) = \underline{(-5, 10)}$

Take $z = (0,1)$ find $z \cdot z = z^2$:

$(0,1)\cdot(0,1) = (-1,0)$

$z^2 = -1$

Wow!!!

Panel 8

There is a (complex) number $z = (0,1)$ with $z^2 = -1$!

Compute: $(x,0) + (0,1)(y,0) =$

$(x,0) + (0,y) = (x,y)$

$x + (0,1)y = (x,y)$

Definition: $(0,1) = i$ (definition) $\Rightarrow i^2 = -1$

Theorem: Every complex number $z = (x,y)$ can be written as $z = x + iy$

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Now everything makes sense:

Ex: For $z_1 = (1,2)$ and $z_2 = (3,4)$, find $z_1 \cdot z_2$:

$z_1 = 1 + 2i, \quad z_2 = 3 + 4i$

$z_1 \cdot z_2 = (1+2i)(3+4i) = 3 + 6i + 4i + 8i^2 =$

$= 3 - 8 + 10i = -5 + 10i =$

$= \underline{\underline{(-5, 10)}}$

Panel 10

Ex: Find $Re(z)$ and $Im(z)$ if

a) $z = i$

$Re(z) = 0$

$Im(z) = 1$

b) $z = (1+i)^2 = 1 + 2i + i^2 = 1 - 1 + 2i = 2i$

$Re((1+i)^2) = 0$

$Im((1+i)^2) = 2$

c) $z = 1/i$ $\qquad \frac{1}{i} \cdot \frac{i}{i} = -i$

$Re(1/i) = 0, \quad Im(1/i) = -1$

Panel 11

Basic Properties

All standard rules apply

Ex: Find i^2, i^3, i^4, i^5, and i^{109}

i

$i^2 = -1$

$i^3 = i\,i^2 = -i$

$i^4 = i^2\,i^2 = (-1)(-1) = 1$

$i^5 = i$

$i^6 = -1$

$i^7 = -i$

$i^{109} = i\,i^{108} = \underline{\underline{i}}$

Panel 12

Solve $5z + 9 = 1$ and $3z = 1$

$\underline{\underline{z = -8/5}}$ $\quad z = 1/3$

Solve $(1+2i)z = 1$ $\quad z = x + iy$

$(1+2i)(x+iy) = 1$

$\underbrace{x + iy + 2ix - 2y}_{} = 1$

$Re(\quad) = x - 2y = 1$

$Im(\quad) = y + 2x = 0$

$x - 2y = 1 \Rightarrow x + 4x = 1$

$y + 2x = 0 \Rightarrow y = -2x$

$\Downarrow$

$x = 1/5$

$y = -2/5$

Panel 13

$(1+2i)\,z = 1 \Rightarrow x = 1/5,\ y = -2/5$ or

$z = \frac{1}{5} - \frac{2}{5}i$

→ single equation in complex

⇔ system of two reals

Def: $(1+2i)z = 1$

$$z = \frac{1}{1+2i} \cdot \frac{(1-2i)}{(1-2i)} = \frac{1-2i}{1+4} = \underline{\underline{\frac{1}{5} - \frac{2}{5}i}}$$

Panel 14

Try these: $\mathrm{Re}(iz) = -y$

$\neq x$

a) $\mathrm{Im}(iz)$ and $\mathrm{Re}(1/i) = 0$ b) $(x+iy)^2 = x^2 - y^2 + 2ixy$

$\mathrm{Im}(1/i) = -1$

b) Show that $1+i$ solves $z^2 - 2z + 2 = 0$

$(1+i)^2 - 2(1+i) + 2 = 0$

$1 + 2i + i^2 - 2 - 2i + 2 = 0$

$\cancel{1} + \cancel{2i} - \cancel{1} - \cancel{2} - \cancel{2i} + \cancel{2} = 0$ ✓

Panel 15

c) Find $\mathrm{Re}\left(\frac{1+2i}{3+4i}\right)$ and $\mathrm{Im}\left(\frac{1+2i}{3+4i}\right)$

$$\left(\frac{1+2i}{3+4i}\right)\cdot\left(\frac{3-4i}{3-4i}\right) = \frac{3-4i+6i+8}{9+16}$$

$$= \frac{11+2i}{25}$$

$$\mathrm{Re}\left(\frac{1+2i}{3+4i}\right) = \frac{11}{25}$$

$$\mathrm{Im}\left(\frac{1+2i}{3+4i}\right) = \frac{2}{25}$$

d) Solve $z^2 = i$ i.e. $\sqrt{i}$

$(x+iy)^2 = i$

$x^2 - y^2 + 2ixy = i \rightarrow x^2 - y^2 = 0 \Rightarrow x^2 = y^2 \Rightarrow x = \pm y$

$2xy = 1 \Rightarrow 2x^2 = 1 \Rightarrow x = \pm 1/\sqrt{2}$

$-2x^2 = 1 \Rightarrow x = \times$

$\sqrt{i} = \frac{1}{\sqrt{2}}(1+i)$ or $-\frac{1}{\sqrt{2}}(1+i)$

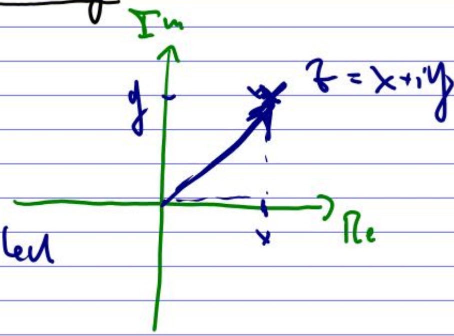
Panel 16

Check $\left[\frac{1}{\sqrt{2}}(1+i)\right]^2 = \frac{1}{2}(1+i)^2 = \frac{1}{2}(1-1+2i) = i$ ✓

Panel 17

Complex Numbers Graphically

$z = (x,y) = x + iy$



Every $z \in \mathbb{C}$ is represented by a 2D vector in $\mathbb{R}^2$

Def $\|z\| = \|x+iy\| = \sqrt{x^2+y^2}$ is called length or norm of z

Panel 18

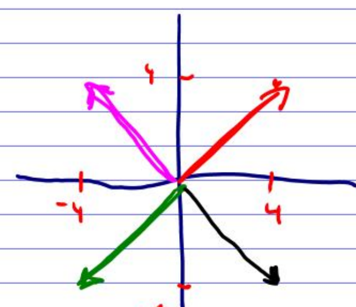
Find $\|1+i\| = \sqrt{2}$

$\|4+4i\| = 4\,\|1+i\| = 4\sqrt{2}$

$\|-4-4i\| = 4\sqrt{2}$

$\|4-4i\| = 4\sqrt{2}$

$\|-4+4i\| = 4\sqrt{2}$



Which one is smaller: $z = 1+3i$ or $w = 2+2i$?

Neither: Complex #'s are not ordered