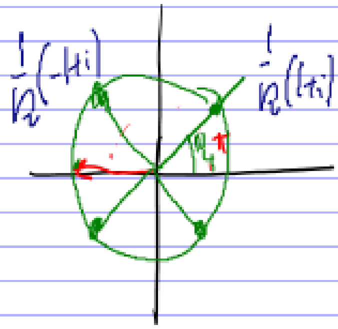
Panel 1

Review

$\sqrt[4]{-1}$



$$\sqrt[5]{1+i} = \sqrt[5]{\sqrt{2}\, e^{i\pi/4}} = 2^{1/10}\, e^{i\left(\frac{\pi}{20} + \frac{2k\pi}{5}\right)}$$

$f(z) = iz - 1$, $z = t + i2t$, $f(t+i2t) = i(t+i2t) - 1 \ldots$

$y = 2x$

1

Panel 2

$$\lim_{z\to 0} f(z) = f(0)$$

$$f = \begin{cases} \frac{x+iy}{x-iy} & \text{else} \\ \# & \text{if } (x,y) = 0 \end{cases}$$

$$\lim_{(x,y)\to(0,0)} \frac{x+iy}{x-iy} = \text{d.n.e.}$$

(different limits for $(0,y) \to (0,0)$
$(x,0) \to (0,0)$

$$f(z) = \begin{cases} \frac{z^9 + z - 2i}{z^{15} + i} & \text{else} \\ -\frac{10}{15} & \text{if } z = i \end{cases}$$

$$f(i) = \frac{0}{-i+i}$$

$$\lim_{z\to i} \frac{z^9 + z - 2i}{z^{15} + i} = \lim_{z\to i} \frac{9z^8 + 1}{15 z^{14}} = \frac{10}{-15}$$

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Panel 3

$f(z) = \underbrace{x^3 - 3xy^2}_{u} + i\big(\underbrace{3x^2y - y^3}_{v}\big)$ analytic?

$u_x = 3x^2 - 3y^2 = v_y = 3x^2 - 3y^2$ ✓

$u_y = -6xy = -v_x = 6xy$ ✓

analytic everywhere, (entire)

$\frac{x-iy}{x^2+y^2} =$

$x = \frac{1}{2}(z + \bar{z})$

$y = \frac{1}{2i}(z - \bar{z})$

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Panel 4

$f(z) = x + 3ix^2y - 3xy^2 - iy$

$u = x - 3xy^2 \qquad v(x,y) = 3x^2y - y$

$u_x = 1 - 3y^2 \overset{?}{=} v_y = 3x^2 - 1 \Rightarrow 1 - 3y^2 = 3x^2 - 1$

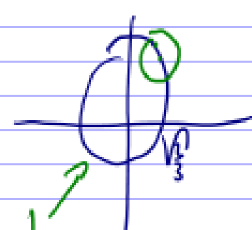
$2 = 3x^2 + 3y^2$

$\frac{2}{3} = x^2 + y^2$

$u_y = -6xy = -v_x = 6xy$

f is $\mathbb{C}$-diff'ble for circle $x^2 + y^2 = 3/2$

but no analytic anywhere!



$f'(z) = u_x + iv_x = x - 3xy^2 + i\,6xy \quad , \forall (x,y)$

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Panel 5

$$u(x,y) = x^3 - 2xy + x^2y^3$$

$$u_x = 3x^2 - 2y + y^3 \quad u_{xx} = 6x$$

not harmonic

$$u_y = -2x + 3xy^2 \quad u_{yy} = 6x$$

$$u_x = 3x^2 - 2y + y^3 \stackrel{?}{=} v_y$$

$$v = 3x^2y - y^2 + \frac{1}{4}y^4 + C(x) \qquad v_x = 6xy + C'(x) = -u_y = 2x - 3xy^2$$

no can do

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Panel 6

$$e^{2+2i} = e^2 e^{2i} = e^2\left(\cos(2) + i\sin(2)\right)$$

$$\cos(\pi + i) = \frac{1}{2}\left(e^{i(\pi+i)} + e^{-i(\pi+i)}\right) = \frac{1}{2}\left(e^{i\pi}e^{-1} + e^{-i\pi}e^{1}\right) =$$

$$= \frac{1}{2}\left(-e^{-1} - e^{1}\right) = -\frac{1}{2}\left(e + e^{-1}\right) =$$

$$= -\cosh(1)$$

$$\mathrm{Log}(1+i) = \ln(\sqrt{2}) + i\frac{\pi}{4}$$

$$\underset{\log}{\mathrm{Log}}(z) = \underset{\log}{\mathrm{Log}}(Re^{it}) = \ln(R) + i\underset{(t + k \cdot 2k\pi)}{(t)}$$

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Panel 7

$\sin(t) = 3i$

$\frac{1}{2i}(e^{it} - e^{-it}) = 3i$

$e^{it} - e^{-it} = -6$ $\quad e^{ix}e^{-y} - e^{-ix}e^{y} = -6$?

$e^{it}(1 - e^{-2it}) = -6$

Hw

$x = 0$

$e^{4t} = 1 \Rightarrow e^{4x}e^{iky} = 1$ $\quad y = \frac{\pi}{2} + \frac{k\pi}{2} = \frac{\pi}{2}(1+k)$

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Panel 8

$f(z) = \mathrm{Re}(z) = x$

$$\lim_{z\to z_0}\frac{\mathrm{Re}(z) - \mathrm{Re}(z_0)}{z - z_0} = \lim_{(x,y)\to(x_0,y_0)}\frac{x - x_0}{(x-x_0) + i(y-y_0)}$$

let $(x_0, y) \to (x_0, y_0)$: 0 $\quad \frac{0}{(y-y_0)}$

let $(x, y_0) \to (x_0, y_0)$: $\frac{x - x_0}{x - x_0 + i0} \Rightarrow 1$ $\quad \Bigg\}$ no match, no limit

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Panel 9

u and v are harmonic conj. $\Rightarrow$ $u_x = v_y$

$u_{xx} + u_{yy} = 0$, $v_{xx} + v_{yy} = 0$ $u_y = -v_x$

$\frac{\partial}{\partial x} u \cdot v = u_x v + u v_x$

$\frac{\partial^2}{\partial x^2} = u_{xx} v + u_x v_x + u_x v_x + u v_{xx} = (uv)_{xx}$

$(uv)_{yy}$

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Panel 10

De Moivre $\left(e^{it}\right)^n = e^{int}$

$\left(\cos(t) + i\sin(t)\right)^n = \cos(nt) + i\sin(nt)$ set $n=2$

$\sin(2x) = ?\ \mathrm{Im}\left(\left[\cos(t) + i\sin(t)\right]^2\right)$

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Panel 11

$$\int_\gamma \frac{1}{\bar{z}}\,dz \qquad \gamma: \text{circle, radius } 2, \text{ center } (0,0)$$

$$z(t) = 2e^{it}, \quad t \in [0, 2\pi]$$

$$\int_0^{2\pi} \frac{1}{(\overline{2e^{it}})} 2ie^{it}\,dt = \qquad \frac{dz}{dt} = i2e^{it}$$

$$\int_0^{2\pi} \frac{2i}{2}\left(e^{it} e^{it}\right) dt = i\int_0^{2\pi} e^{2it}\,dt = i\frac{1}{2i} e^{2it}\Big|_0^{2\pi} = \frac{1}{2}\left(e^{4\pi i} - e^{0}\right) = 0$$

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