Panel 1

Complex HW #7

① Show that the following functions are not $\mathbb{C}$-diffble:

a) $f(z) = 2x + ixy^2$ b) $f(z) = z - \bar{z}$

c) $f(z) = e^x e^{-iy}$ Hint: check CR equations

② Use the CR equations to show that $f'(z)$ exists if $f(z) = z^3$ and verify that $f'(z) = 3z^2$

③ Let $f(z) = x^3 + i(1-y)^3$. Show that f is $\mathbb{C}$-diffble only for $z = i$ and find $f'(z)$

④ Suppose $f(z)$ is $\mathbb{C}$-diffble in a domain D and $f(z)$ is real-valued. Prove that $f(z)$ must be constant. (Real-valued means that $f(z) \in \mathbb{R}$ $\forall z \in D$) Hint: CR

1

Panel 2

⑤ Prove that every finite set in $\mathbb{C}$ is closed.

Hint: You could try to show that a finite set has no accumulation points, then use a previous HW problem.

⑥ Find a (infinite) sequence of points that is a closed set.

Hint: Try a sequence that has no accumulation points

⑦ Verify the CR equation in polar coordinates (slide 18 from last lecture)

2