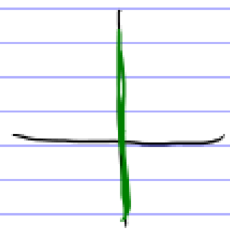
Panel 1

$$f(z) = \frac{\bar{z}}{z+\bar{z}}, \quad z+\bar{z} = 2x = 0 \iff x = 0$$

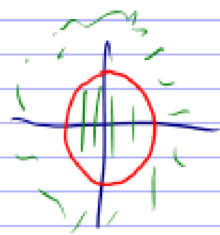


$$\mathbb{C} - \{x=0\}, \quad Re(z) = x \neq 0$$

$$\mathbb{C} - \{0, y\}$$

$$f(z) = \frac{\quad}{1 - z\bar{z}}$$

$$1 = z\bar{z} = |z|^2 \iff |z| = 1$$



$$\mathbb{C} - \{|z| = 1\}$$

all $z : |z| \neq 1$

1

Panel 2

$$f(z) = f(re^{i\theta}) = f(r\cos\theta + i r \sin\theta) =$$

$$(re^{i\theta})^2 = (r\cos\theta + i r\sin\theta)^2$$

$$r^2 e^{i2\theta} = \underbrace{r^2\cos(2\theta)}_{u} + i\underbrace{r^2\sin(2\theta)}_{v}$$

2

Panel 3

Mapping Properties of $f(z) = e^z$ for

a) horizontal lines

b) vertical lines

3

Panel 4

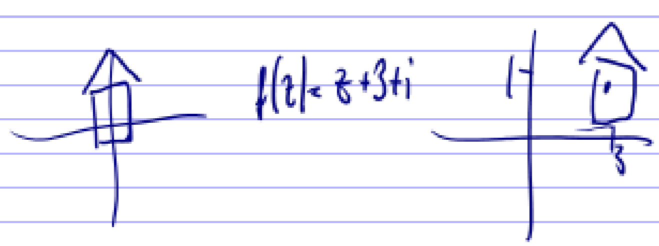
Mapping Properties of Linear Functions

$f(z) = az$ with $|a| = 1$ is a rotation by t, $a = e^{it}$: $f(z) = e^{it} z = e^{it} r e^{i\theta} = r e^{i(\theta + t)}$

$f(z) = az$ is a rotation and stretch/shrink

$f(z) = z + b$ is a translation by $b = b_1 + ib_2$

$f(z) = az + b$ is a: rotation, stretch/shrink + translation

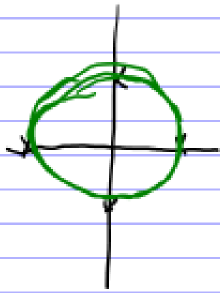


4

Panel 5

Ex: Consider the elipse $\gamma(t) = \underbrace{2\cos(t) + i\sin(t)}$. Rotate it by $\pi/6$, shift it up by 1 and 2 to the right. Verify using Maple.

$$f(z) = e^{i\pi/6} + 2+i$$



$$f(2\cos(t) + i\sin(t)) = e^{i\pi/6}(2\cos(t)+i\sin(t)) + 2+i =$$

$$= (\cos(\tfrac{\pi}{6}) + i\sin(\tfrac{\pi}{6}))(2\cos(t) + i\sin(t)) + 2+i$$

$$2\cos(\tfrac{\pi}{6})\cos(t) - \sin(\tfrac{\pi}{6})\sin(t) + 2 + i(2\sin(\tfrac{\pi}{6})\cos(t) + \cos(\tfrac{\pi}{6})\sin(t) + 1)$$

5

Panel 6

We understand complex numbers "individually"

$\Rightarrow$ move on to collections of numbers: Topology

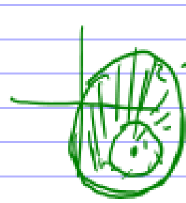
Want to describe sets of points in $\mathbb{C}$:

$|z| = 2$ $\Leftrightarrow$ circle, center 0, radius 2

$|z - 3 + 4i| = 2$ $\Leftrightarrow$ circle, center $(3-4i)$, radius 2

$|z - 3 + 4i| < 2$ $\Leftrightarrow$ disk —"—

$1 < |z - 3 + 4i| < 2$



doughnut

annulus

6

Panel 7

Definition: $D_r(z_0) = \{z \in \mathbb{C} : |z - z_0| < r\}$ is open disk centered at z_0, radius r.

ε- neighborhood of z_0 is a "small" $D_\varepsilon(z_0)$

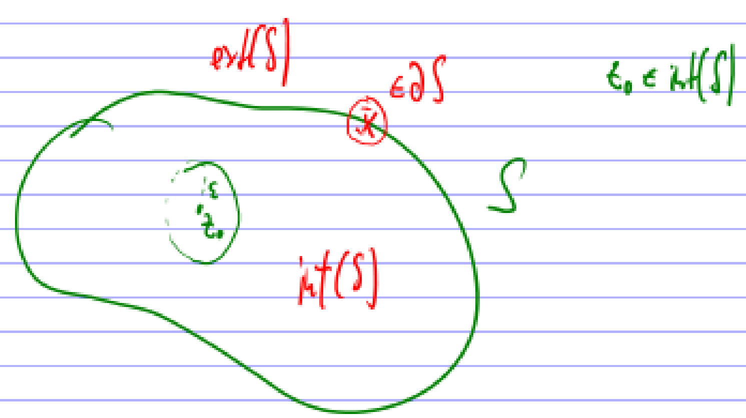
Interior of a set S : $z \in \text{int}(S)$ if there is an ε-nbhd of z contained in S

Boundary of a set S : $z \in \partial S$ if every ε-nbhd of z contains points for S and outside S

7

Panel 8

Def: ext(S) is exterior of S is



8

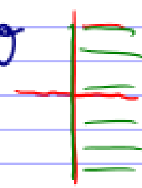
Panel 9

Def: A set $D \subset \mathbb{C}$ is open if every point is an interior point.

A set $D \subset \mathbb{C}$ is closed if it contains all of its bdry points.

Ex: $D_r(z_0) = \{|z-z_0|<r\}$ open

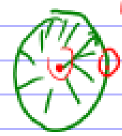
$Re(z) \geq 0$



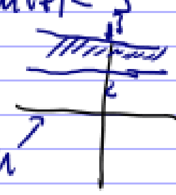
$|z| \leq 2$ closed

neither

$0 < |z-1| \leq 1$



$2 < Im(z) < 3$



$\mathbb{C}$ open + closed

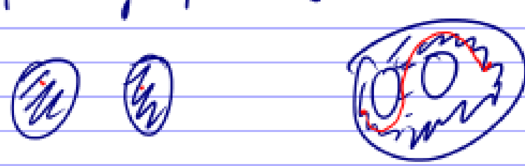
$\emptyset$ = open + closed

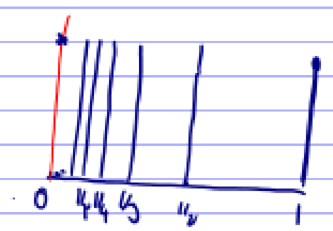
open

9

Panel 10

Definition: A set S is connected if every $z_1, z_2 \in S$ can be joined by a path in S.





connected or not?

10

Panel 11

Def: A point z_0 is called accumulation point of a set S if for every ε the punctured disk $D_\varepsilon(z_0)$ (without z_0) contains at least one point of S

Ex: $D_1(0)$ ← accumulation points of S (don't have to be part of S)

Ex: $\{1, 1/2, 1/3, 1/4, \dots\}$