Panel 1

Last time:

$|z - z_0| = R$

$z \in \mathbb{C}$ as vectors

$|z + i| = |z - (-i)| = R$

$\Uparrow$

$\|z\| =$

$\bar{z} =$

arg(z) is angle of z

Arg(z) is angle $-\pi < \theta \le \pi$

Euler's Formula: $e^{it} = \cos(t) + i\sin(t)$

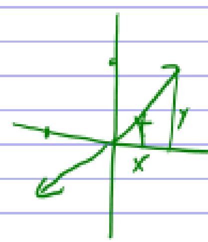
$z = Re^{it}$

To multiply z, w: mult. length, add angles

1

Panel 2

$$z = \frac{i}{-2-2i} \cdot \frac{(-2+2i)}{(-2+2i)}$$



$t = \arctan\left(\frac{y}{x}\right)$

$$z = \frac{1e^{i\frac{\pi}{2}}}{\sqrt{8}\, e^{i\frac{3\pi}{4}}} = \frac{1}{\sqrt{8}} e^{i\left(\frac{\pi}{2} - \frac{3\pi}{4}\right)} =$$

$$= \frac{1}{\sqrt{8}} e^{i\left(-\frac{\pi}{4}\right)}$$

$\text{Arg}(\ ) = -3\pi/4$

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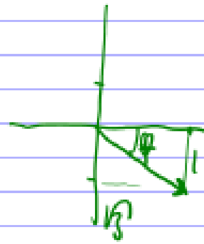
Panel 3

$$z = \left(\sqrt{3} - i\right)^6$$

$$= \left[2e^{-i\frac{\pi}{6}}\right]^6$$



$$\varphi = \arctan\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$$

$$= 2^6 e^{-\pi i} = -2^6$$

$\mathrm{Arg}(z) = \pi$

3

Panel 4

Note: $\left(e^{i\theta}\right)^n = \left(\cos(t) + i\sin(t)\right)^n = \cos(nt) + i\sin(nt)$

De Moivre

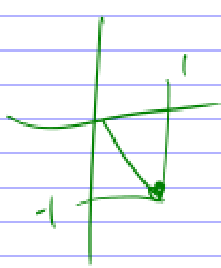
$n = 2 \Rightarrow$ trig. id.

$$e^{-\frac{3\pi}{2}i} = i$$

$$2e^{i2\pi} = 2$$

$$3e^{-\frac{\pi}{4}i} = \frac{3}{\sqrt{2}}(1-i)$$

$$4e^{-\frac{\pi}{2}i} = -4i$$



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Panel 5

Note: If $z_1 = r_1 e^{i\theta_1}$ and $z_2 = r_2 e^{i\theta_2}$ $\Rightarrow$ $z_1 z_2 = r_1 r_2 e^{i(\theta_1+\theta_2)}$

Thm: $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$

Ex: $z_1 = -1$, $z_2 = i$ $\Rightarrow$ $z_1 z_2 = -i$

$\arg(z_1) = \pi$, $\arg(z_2) = \pi/2$ $\qquad \arg(z_1 z_2) = \frac{3\pi}{2}$

Note: $\mathrm{Arg}(z_1 z_2) \neq \mathrm{Arg}(z_1) + \mathrm{Arg}(z_2)$

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Panel 6

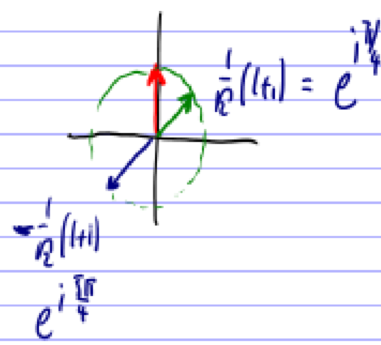
Complex Roots

$\sqrt{i}$

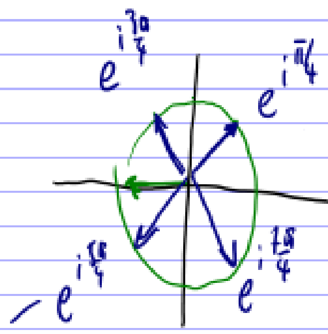
$(e^{i\pi/4})^2 = e^{i\pi/2} = i$ ✓

$(e^{i\frac{5\pi}{4}})^2 = e^{i(\frac{5\pi}{2})}$

$= e^{i(2\pi + \frac{\pi}{2})} = e^{i2\pi} e^{i\pi/2} = i$



$\sqrt[4]{-1}$



$(e^{i\frac{5\pi}{4}})^4 = e^{i5\pi} = e^{i\pi} = -1$

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Panel 7

To find n-th root of a: $z^n = a$

$$z^n = re^{i\theta} = re^{i(\theta+2\pi)} = re^{i(\theta+4\pi)} \cdots re^{i(\theta+2\pi n)}$$

$$z = r^{1/n}e^{i\frac{\theta}{n}},\quad r^{1/n}e^{i\left(\frac{\theta}{n}+\frac{2\pi}{n}\right)},\quad r^{1/n}e^{i\left(\frac{\theta}{n}+\frac{4\pi}{n}\right)},\quad r^{1/n}e^{i\left(\frac{\theta}{n}+2\pi\right)}$$

Thm: The n-th roots of $a = re^{i\theta}$ are

$$z_k = r^{1/n}e^{i\left(\frac{\theta}{n}+\frac{2\pi k}{n}\right)},\quad k=0,1,\dots,n-1$$

Corollary: There are n distinct n-th roots of $a\in\mathbb{C}$, $a\neq 0$

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Panel 8

Ex: Find all 4^{th} roots of i

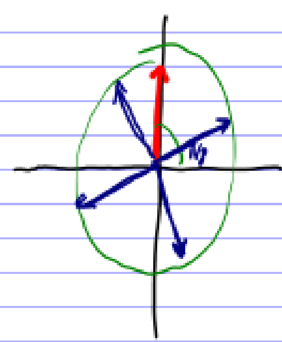
$i = 1e^{i\pi/2}$: $\sqrt[4]{i} = e^{i\pi/8}$

$e^{i\left(\frac{\pi}{8}+\frac{2\pi}{4}\right)}$

$e^{i\left(\frac{\pi}{8}+\frac{4\pi}{4}\right)}$

$e^{i\left(\frac{\pi}{8}+\frac{6\pi}{4}\right)}$

$e^{i\left(\frac{\pi}{8}+\frac{\cancel{8\pi}}{4}\right)}$ 7π



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Panel 9

Find $\sqrt[4]{z}$ graphically

① Draw ~~unit~~ circle including z circle radius $|z|$

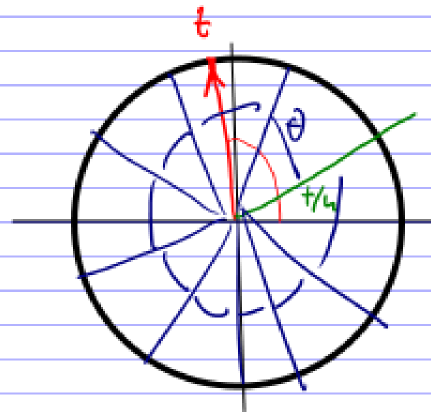
② Divide angle by 4

(2a) Length is $\sqrt[4]{|z|}$

③ Draw regular polygon

with 4 corners

④ Adjust the radius



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Panel 10

http://math.mit.edu/daimp/ComplexRoots.html

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Panel 11

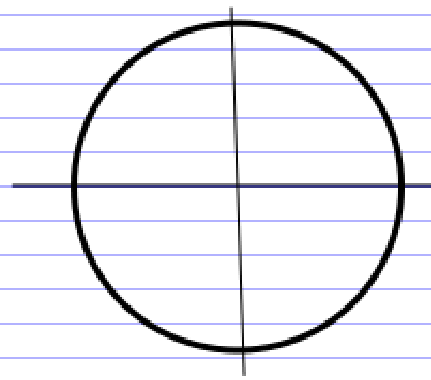
Find $\sqrt[3]{-8i}$ graphically (use prev. applet)

① Draw unit circle and $-8i$, approx

② Divide angle by ___

③ Draw regular polygon with ___ corners

④ Adjust the radius



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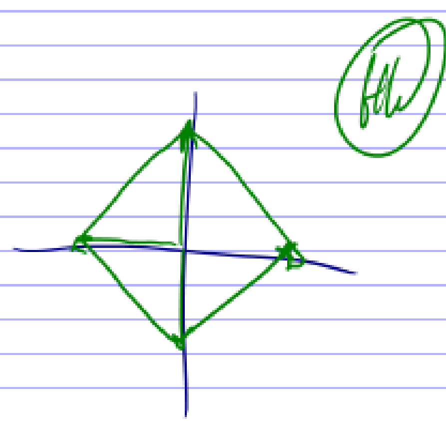
Panel 12

Theorem: If $\omega_k = e^{i\frac{2k\pi}{n}}$, $k = 0, 1, \ldots, n-1$ are the n-th roots of unity then:

$$1 + \omega_1 + \omega_2 + \omega_3 + \ldots + \omega_{n-1} = 0$$



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