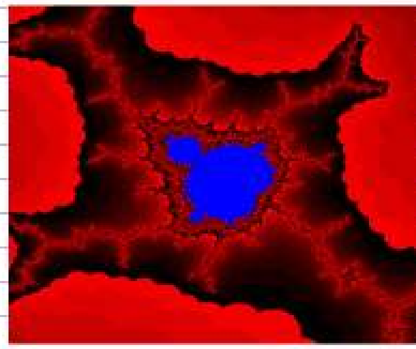
Panel 1

Welcome to Math 4512

Complex Analysis



(where $z^2+1=0$ does have solutions
and numbers are no longer sorted)

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Panel 2

Math 4512: Complex Analysis

Instructor: Bert Wachsmuth

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SC118D (through 1/6), MW 3-4pm

Grading:
2 exams (45%)
HW+quizzes (45%)
Participation (10%)

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Panel 3

Other Stuff: http://pirate.shu.edu/~wachsmut/

Dyknow - see web page.

Material covered: Chapters 1 to 6 + extras

Complex #'s and algebra
Complex functions: limits, cont.,
Differentiation + Integrations
Series
Residues + Applications

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Panel 4

Dyknow: Note-taking and Collaboration Tool

- download the program
- double-click on downloaded software to install

You will be prompted for some information to enter. Use the following:

- for "*DyKnow Server Address*", use: `dyknow://vision.dyknow.com/shu.edu`

Leave everything else as it is. In addition, you will need a user name and password, which you will receive from your instructor.

User name: 8-letters

Password: – " –

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Panel 5

Definition of Complex Numbers:

Pairs $z = (x, y)$ where x, y are real, and the sum and product defined as:

$$z_1 = (x_1, y_1) \quad \text{and} \quad z_2 = (x_2, y_2)$$

$$z_1 + z_2 = (x_1 + x_2, y_1 + y_2)$$

$$z_1 \cdot z_2 = (x_1, y_1) \cdot (x_2, y_2) = (x_1 x_2 - y_1 y_2, x_1 y_2 + x_2 y_1)$$

Sometimes we write $\vec{0}$ for $(0,0)$

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Panel 6

$z_1 + z_2 = (x_1, y_1) + (x_2, y_2) =$ ✓ makes sense

$z_1 \cdot z_2 = (x_1, y_1) \cdot (x_2, y_2) = (x_1 x_2 - y_1 y_2, x_1 y_2 + x_2 y_1)$ ~ why?

Why not. $z_1 \cdot z_2 = (x_1, y_1) \cdot (x_2, y_2) \overset{?}{=} (x_1 x_2, y_1 y_2)$

<u>Want:</u> $z_1 \cdot z_2 = 0 \Rightarrow z_1 = 0$, or $z_2 = 0$

$\underset{\neq \vec{0}}{(1,0)} \cdot \underset{\neq \vec{0}}{(0,5)} = (0,0) = \vec{0}$ Bad Definition

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Panel 7

$z_1 + z_2 = (x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$

$z_1 \cdot z_2 = (x_1, y_1) \cdot (x_2, y_2) = (x_1 x_2 - y_1 y_2, x_1 y_2 + x_2 y_1)$

What if $z_1 = (\underline{x_1}, 0)$ and $z_2 = (\underline{x_2}, 0)$?

$\Rightarrow z_1 + z_2 = (x_1 + x_2, 0)$

and $z_1 \cdot z_2 = (x_1 x_2, 0)$

Thus: $\mathbb{R} \subset \mathbb{C}$

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Panel 8

<u>What's the Deal (I mean, really)?</u>

Take $z_1 = (1,2)$ and $z_2 = (3,4)$, find $z_1 \cdot z_2$.

$z_1 \cdot z_2 = (1,2) \cdot (3,4) = (1 \cdot 3 - 2 \cdot 4, 1 \cdot 4 + 2 \cdot 3) = (-5, 10)$

$z_1 + z_2 = (4,6)$, $z_1 - z_2 = (-2,-2)$, z_1 / z_2 ?

Take $z = (0,1)$, find

$z \cdot z = (0,1) \cdot (0,1) = (-1, 0)$

$z^2 = \underline{\underline{-1}}$

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Panel 9

There is a (complex) number $z = (0,1) = i$ (by def) with $z^2 = -1$!

Compute: $(x,0) + (0,1)(y,0) = (x,0) + (0-0, 0+y) = (x,y)$

$x + i\,y = (x,y) = z$

<u>Definition:</u> $i = (0,1) \Rightarrow z^2 = -1$

<u>Theorem:</u> Every complex number $z = (x,y)$ can be written as $z = x + iy$ where

$x = Re(z)$, $y = Im(z)$, and $i^2 = -1$

Real part   Imaginary part

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Panel 10

Now everything makes sense:

<u>Ex:</u> For $z_1 = (1,2)$ and $z_2 = (3,4)$, find $z_1 \cdot z_2$:

$(1,2) \cdot (3,4) =$

$(1+i2) \cdot (3+i4) = 1 \cdot 3 + 4i + 6i + 8i^2 =$

$= 3 - 8 + i(4+6) = (-5, 10) = -5 + 10i$

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Panel 11

Ex: Find $Re(z)$ and $Im(z)$ if

a) $z = i = 0 + 1\cdot i$, $Re(i) = 0$
$Im(i) = 1$

b) $z = (1+i)^2 = 1 + 2i + i^2 = 2i$

$Re(z) = 0$
$Im(z) = 2$

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Panel 12

Basic Properties

All standard rules apply – associative, distr. law, commutative

Ex: Find i^2, i^3, i^4, i^5, and i^{109}

$i = i$
$i^2 = -1$
$i^3 = i \cdot i^2 = -i$
$i^4 = i^2 i^2 = 1$
$i^5 = i$
$i^6 = -1$

$i^{109} = i^{108}\, i = 1 \cdot i = i$

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Panel 13

Prove that $z_1 \cdot z_2 = z_2 \cdot z_1$

$z_1 \cdot z_2 = (x_1, y_1) \cdot (x_2, y_2) = (x_1 x_2 - y_1 y_2, x_1 y_2 + x_2 y_1)$

$z_2 \cdot z_1 = (x_2, y_2)(x_1, y_1) = (x_2 x_1 - y_2 y_1, x_2 y_1 + y_2 x_1)$

same by rules in $\mathbb{R}$

HW $z_1 \cdot (z_2 + z_3) = z_1 z_2 + z_1 z_3$

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Panel 14

Solve $5z + 9 = 1$ and $3z = 1$

$5z = -8$ $z = 1/3$

$z = -8/5$

Solve $(1+2i) z = 1$

$(1+2i)(x+iy) = 1$

$x - 2y + i(2x+y) = 1$ $\Leftrightarrow$ $x - 2y = 1$, $2x + y = 0$; $5x = 1$, $y = -2x$; $x = 1/5$, $y = -2/5$

Or: $z = \frac{1}{1+2i} \cdot \frac{1-2i}{1-2i} = \frac{1-2i}{5} = \frac{1}{5} - i\frac{2}{5}$

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Panel 15

Try these: $Re(iz) = -y$ $\frac{1}{i} \cdot \frac{i}{i} = \frac{i}{-1} = -i$

a) $Im(iz)$ and $Re(1/i) = 0$ b) $(x+iy)^2 = x^2 + i2xy - y^2$

$Im(i(x+iy)) = x$ $Im(1/i) = -1$

b) Show that $1+i$ solves $z^2 - 2z + 2 = 0$

$2i - (2+2i) + 2 = 0$

c) Find $Re\left(\frac{1+2i}{3+4i}\right) = \frac{11}{25}$ and $Im\left(\frac{1+2i}{3+4i}\right) = -\frac{2}{25}$

$\frac{1+2i}{3+4i} \cdot \frac{(3-4i)}{(3-4i)} = \frac{3-4i+6i-8i^2}{9+16} = \frac{3+8+i(6-4)}{25} = \frac{11-2i}{25}$ $\sqrt{-1} = i$

d) Solve $z^2 = i$ i.e. $\sqrt{i}$

$x^2 - y^2 + 2ixy = i$ $Re(z^2) = x^2 - y^2$

$\Rightarrow x^2 - y^2 = 0,\ 2xy = 1,\ 2x^2 = 1,\ x = \pm\frac{1}{\sqrt{2}},\ y = \pm\frac{1}{\sqrt{2}}$

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Panel 16

Thus: $\sqrt{i} = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$ or

$-\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$

HW: $\sqrt{1+i}$

$\sqrt{i} = \pm\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$

Note: $z_1/z_2 = \frac{x_1+iy_1}{x_2+iy_2} \frac{(x_2-iy_2)}{(x_2-iy_2)} = \frac{x_1x_2 + y_1y_2 + i(y_1x_2 - x_1y_2)}{x_2^2 + y_2^2}$

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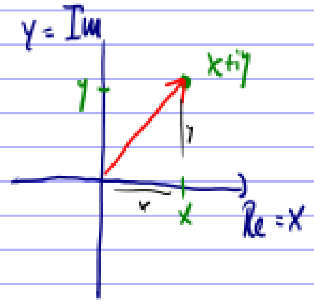
Panel 17

Complex Numbers Graphically

$z = (x, y) = x + iy$



every $z \in \mathbb{C}$ is a point in $\mathbb{R}^2$
or vector in $\mathbb{R}^2$

Def: Length of z is $\|z\| = \sqrt{x^2+y^2}$

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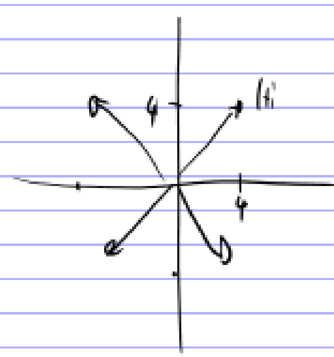
Panel 18

Find $|1+i| = \sqrt{2}$

$|4+4i| = \sqrt{32}$

$|-4-4i| = \sqrt{32}$

$|4-4i| = \sqrt{32}$



Which one is smaller: $z = 1+3i$ or $w = 2+2i$?

Can not tell! Complex # are not ordered!

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Panel 19

$\sqrt{x^2+y^2} = 2 \Rightarrow$

Describe the set of all z s.t. $|z| = 2$

Circle radius 2, center 0.

$x^2+y^2=4$

$(2\cos(t), 2\sin(t))$

$\underline{|z|=2}$

$|z-(3+i)|=1$

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