Panel 1

A Note about i^i and Friends

① $i = \sqrt{-1}$ is the "imaginary unit"

② $i^i = e^{\log(i^i)} = e^{i \log(i)} = e^{i(i(\pi/2 + 2k\pi))} = e^{-\pi/2} e^{-2k\pi}$

Taking the principle part we have $i^i = e^{-\pi/2}$. Note in particular that i^i is a real number!

Let's confirm this using Maple (where you must pick i from the list of symbols)

evalc(i^i)

$e^{-\frac{1}{2}\pi}$

1

Panel 2

③ i^{i^i} : There could be three interpretations:

a) $i^{i^i} = i^{i \cdot i} = i^{-1} = 1/i$ because "powers to powers" multiply

b) $(i^i)^i = [e^{-\pi/2} e^{-2k\pi}]^i = e^{-\pi/2 i} e^{-2k\pi i} = -i \cdot 1 = -i = 1/i$

c) $i^{(i^i)} = i^{e^{-\pi/2} e^{-2k\pi}} = e^{e^{-\pi/2} e^{-2k\pi} \cdot \log(i)} = e^{e^{-\pi/2} e^{-2k\pi} i(\pi/2 + 2k\pi)}$

$= e^{i\pi/2 \, e^{-\pi/2} e^{-2k\pi}} e^{i 2k\pi e^{-\pi/2} e^{-2k\pi}} =$

$= e^{i\pi/2 \, e^{-\pi/2} e^{-2k\pi}} \cdot (e^{i 2k\pi})^{e^{-\pi/2} e^{-2k\pi}} =$

$= e^{i\pi/2 \, e^{-\pi/2} e^{-2k\pi}} \cdot 1$

2

Panel 3

$$i^{(i^i)} = e^{i\,\pi/2\, e^{-\pi/2} e^{-2k\pi}} =$$

$$= \cos\left(\tfrac{\pi}{2} e^{-\pi/2} e^{-2k\pi}\right) + i \sin\left(\tfrac{\pi}{2} e^{-\pi/2} e^{-2k\pi}\right)$$

Taking the principle part only we get:

$$i^{(i^i)} = \cos\left(\pi/2\, e^{-\pi/2}\right) + i \sin\left(\pi/2\, e^{-\pi/2}\right).$$

Confirm with Maple:

$evalc\left(i^{i^i}\right)$

$$\cos\left(\frac{1}{2} e^{-\frac{1}{2}\pi}\pi\right) + I \sin\left(\frac{1}{2} e^{-\frac{1}{2}\pi}\pi\right)$$

3

Panel 4

So we worked out two possibilities (principle part only)

$$i^{i^i} = \begin{cases} (i^i)^i = -i = 1/i & \text{A} \\ e^{i\pi/2\, e^{-\pi/2}} = \cos\left(\frac{\pi}{2} e^{-\pi/2}\right) + i \sin\left(\pi/2\, e^{-\pi/2}\right) & \text{B} \end{cases}$$

Which one is right? Well, think about 3^{3^3}...

We will take a vote on Wed in class...

4