

Panel 1

Complex HW

① Show that  $u(x,y)$  is harmonic and find harm. conjugate:

a)  $u(x,y) = 2x - x^3 + 3xy^2$

b)  $u(x,y) = \sinh(x) \sin(y)$

c)  $u(x,y) = \frac{y}{x^2+y^2}$

② Prove that if  $v(x,y)$  and  $V(x,y)$  are both harmonic conjugates of  $u(x,y)$  then  $v$  and  $V$  can differ by at most a constant.

Panel 2

③ We defined  $e^z = e^x e^{iy} = e^x (\cos(y) + i \sin(y))$

a) solve  $e^z = 1-i$  and  $e^z = -2$  (all solutions)

b) show that  $e^z$  is entire but  $e^{\bar{z}}$  is nowhere  
diffble

c) show that  $|e^{-z}| < 1$  iff  $\operatorname{Re}(z) > 0$

④ We defined  $\log(z) = \ln(r) + i(\theta + 2k\pi)$  and

$\operatorname{Log}(z) = \ln(z) + i\theta$ , where  $z = re^{i\theta}$

a) find  $\log(i)$  and  $\log(-3)$

b) show that  $\operatorname{Log}(i^3) \neq 3 \operatorname{Log}(i)$  but

$$\operatorname{Log}(|1-i|^2) = 2 \operatorname{Log}(1-i)$$

here  $\Rightarrow$

Panel 3

⑤ For extra credit, try to determine the values of  
a)  $2^i$       b)  $i^i$       c)  $i^{i^i}$