Panel 1

Review Questions

① Find $f(1+i)$ for (a) $f(z) = z + z^{-1} + 5$ (b) $f(z) = 4z\bar{z}$

② Let $f(z) = z^{21} - 5z^7 + 9z^4$. Find $f(-1+i)$

③ Express in the form $f(z) = u(x,y) + iv(x,y)$ for

(a) $f(z) = z^3$ (b) $f(z) = \bar{z}^2 + (2-3i)z$

④ Let $f(z) = (3+4i)z - 2+i$. Find the image of

(a) the line $x(t) = t$, $y(t) = 1-2t$

(b) the disk $|z-1| < 1$

⑤ Let $f(z) = (2+i)z - 2i$. Find image of the triangle with vertices $z_1 = -2+i$, $z_2 = -2+2i$, $z_3 = 2+i$

Note: Linear functions map lines to lines and circles to circles.

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Panel 2

Make sure to Memorize the following terms

$D_r(z_0) = \{z : |z-z_0| < r\}$ is the open disk centered at z_0 with radius r.

$D_r^*(z_0) = \{z : 0 < |z-z_0| < r\}$ is the deleted or punctured disk $D_r(z_0)$ with the point $z = z_0$ removed

An ε-neighborhood of z_0 is an open disk around z_0 with radius ε

A point $z \in D \subset \mathbb{C}$ is called interior point of D if there is an ε-nbhd. of z containing only points of D

A point $z \in \mathbb{C}$ is called exterior point of D if there is an ε-nbhd. of z containing no points from D

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Panel 3

A point $z \in \mathbb{C}$ is called a boundary point of D if it is neither interior nor exterior point of D

A set $D \subset \mathbb{C}$ is open if: for every point $z \in D$ there is an ε-nbhd. contained in D, i.e. all $z \in D$ are interior points

A set $D \subset \mathbb{C}$ is closed if it contains all of its boundary points.

A set $D \subset \mathbb{C}$ is connected if every two points $z_1, z_2 \in D$ can be connected by a polygon.

A set $D \subset \mathbb{C}$ that is non-empty, open, and connected is called a domain

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Private Freehand 3

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Panel 4

A set $D \subset \mathbb{C}$ is bounded if it is contained in some circle $\{|z| < R\}$. If not it is called unbounded

A point $z_0 \in \mathbb{C}$ is an accumulation point of D if each deleted nbhd. of z_0 contains at least one point of D

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Panel 5

1. Sketch the following sets and determine which are domains:

(a) $|z - 2 + i| \leq 1$; *(b)* $|2z + 3| > 4$;
(c) $\text{Im}\, z > 1$; *(d)* $\text{Im}\, z = 1$;
(e) $0 \leq \arg z \leq \pi/4$ $(z \neq 0)$; *(f)* $|z - 4| \geq |z|$.

2. Which sets in Exercise 1 are neither open nor closed?

3. Which sets in Exercise 1 are bounded?

7. Determine the accumulation points of each of the following sets:

(a) $z_n = i^n$ $(n = 1, 2, \ldots)$; *(b)* $z_n = i^n/n$ $(n = 1, 2, \ldots)$;
(c) $0 \leq \arg z < \pi/2$ $(z \neq 0)$; *(d)* $z_n = (-1)^n(1+i)\dfrac{n-1}{n}$ $(n = 1, 2, \ldots)$.

*8. Prove that if a set contains each of its accumulation points, then it must be a closed set.

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