Panel 1

HW - Complex Analysis

① Find the natural domain of (a) $f(z)=\frac{z}{z+\bar{z}}$, (b) $g(z)=\frac{1}{1-z\bar{z}}$
Describe your answer in geometric terms or draw it. ($z+\bar{z}\neq 0$)

② Every complex function $f(z)=u(x,y)+iv(x,y)$. Find $u(x,y)$ and $v(x,y)$ for $f(z)=z^3+z+1$

③ Suppose $f(z)=x^2-y^2-2y+i(2x-2xy)$. Rewrite the function in terms of z and $\bar{z}$, where $x=\frac{z+\bar{z}}{2}$ and $y=\frac{z-\bar{z}}{2i}$

④ Visit http://www.mathcs.org/java/programs/ZMap/index.html and read "Math Background" and "zMap Quick Guide"

⑤ Answer the first question of the "zMap Sample Questions"

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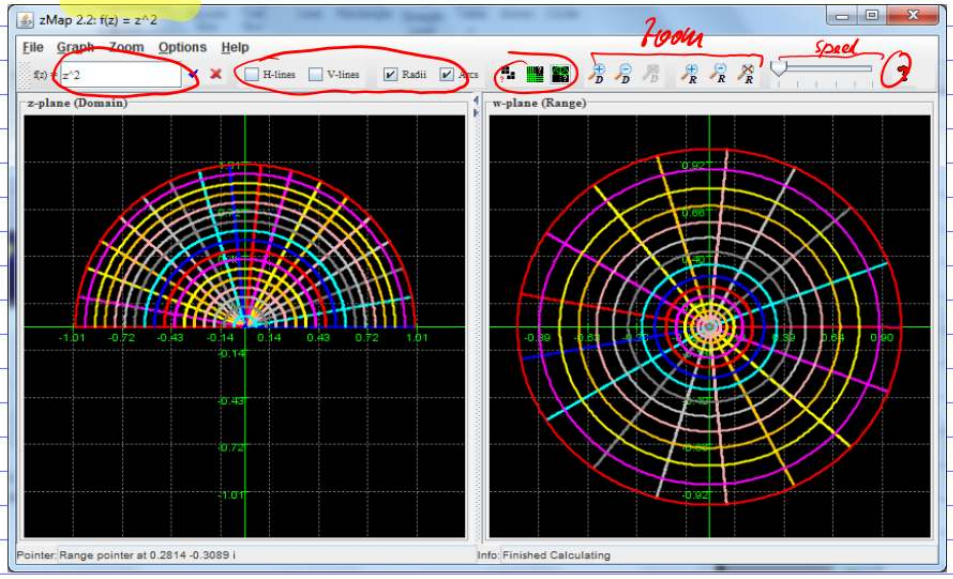
Panel 2

zMap: Program to visualize complex functions



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Panel 3

http://www.mathcs.org/java/programs/ZMap/index.html

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Private Freehand 3

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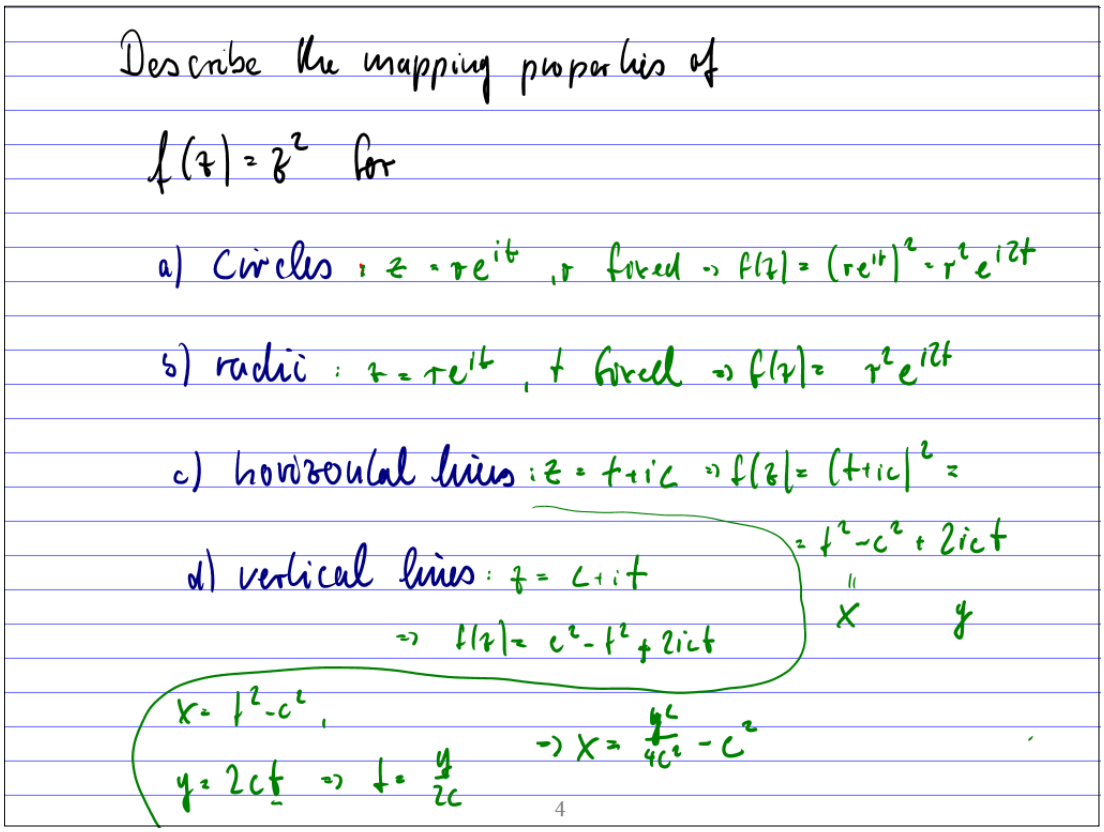
Panel 4



Private Freehand 4

Panel 5

Mapping Properties of $f(z) = e^z$ for

a) horizontal lines : $z = t + ic$: $f(z) = e^{t+ic} = e^t \cdot e^{ic}$

b) vertical lines : $z = c + it$: $f(z) = e^{c+it} = e^c e^{it}$

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Panel 6

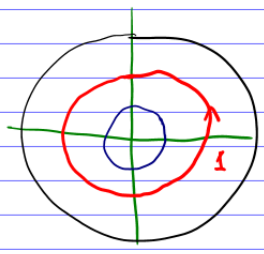
Mapping Properties of $f(z) = 1/z$ for

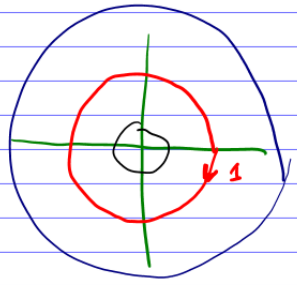
well, for what?

$f(z) = 1/z$. take $z = re^{i\theta}$ $\rightarrow$ $f(z) = (re^{i\theta})^{-1} =$

$= r^{-1}(e^{i\theta})^{-1} =$

$\frac{1}{r} e^{-i\theta}$





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Panel 7

Mapping Properties of Linear Functions

$f(z) = az$ with $|a| = 1$ is a rotation by a fixed angle

$f(z) = az$ is a rotation and shrink ($r<1$) expanding ($r>1$)

$f(z) = z + b$ is a translation

$f(z) = az + b$ is a: rotation, shrink expand, translation!

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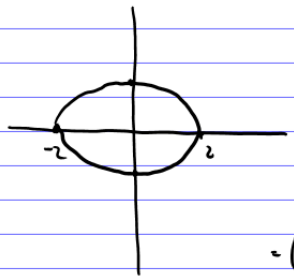
Panel 8

Ex: Consider the ellipse $z(t) = 2\cos(t) + i\sin(t)$. Rotate it by $\pi/6$, shift it up by 1 and 2 to the right. Verify using Maple.



rotate by $\pi/6$, translate by 1 up, 2 right

use $f(z) = e^{i\pi/6} z + 2 + i$

$\Rightarrow e^{i\pi/6}(2\cos(t) + i\sin(t)) + 2 + i =$

$= (\cos(\pi/6) + i\sin(\pi/6))(2\cos t + i\sin(t)) + 2 + i$

($\cos(\pi/6) = \sqrt{3}/2$, $\sin(\pi/6) = 1/2$)

$= \sqrt{3}\cos(t) - \frac{1}{2}\sin(t) + 2 + i\left(\cos(t) + \frac{\sqrt{3}}{2}\sin(t) + 1\right)$

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Panel 9

We understand complex numbers "individually"
⇒ move on to collections of numbers : Topology

Want to describe sets of points in $\mathbb{C}$:

$|z| = 2$ is a circle, center 0, radius 2 $|z-5| = r$

$|z - 3+4i| = 2$ is a circle, centered at $3-4i$, radius 2

$|z - 3+4i| < 2$ is a disk centered at $3-4i$, radius 2

$1 < |z - 3+4i| < 2$ is a washer centered at $3-4i$, radius between 1 and 2

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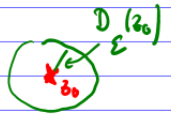
Panel 10

Definition: $D_r(z_0) = \{ z : |z - z_0| < r \}$ is an open disk

$D_r^*(z_0) = \{ z : 0 < |z - z_0| < r \}$ is punctured disk

$\overline{D_r(z_0)} = \{ z : |z - z_0| \leq r \}$ is closed disk

ε-neighborhood of z_0 is $D_\varepsilon(z_0)$
nhbd



Interior of a set S: $z \in \text{int}(S)$ if there is a ε-nhbd contained in S

Boundary of a set S: $z \in \partial S$ if every ε-nhbd of z includes points inside S and outside S

Of
∂X

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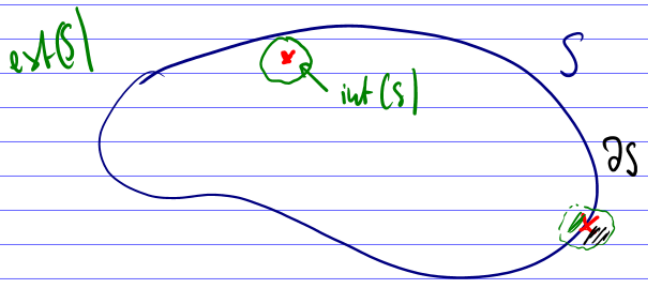
Panel 11



Ex: If $S = D_1(0)$. what is interior, exterior, bdry of S

boundary is unit circle

interior of $D_1(0)$ is itself

exterior is $|z| \geq 1$ (outside unit disk + unit circle)

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Panel 12

Def: A set $D \subset \mathbb{C}$ is <u>open</u> if every point is an interior point

A set $D \subset \mathbb{C}$ is closed if it contains all of its boundary points

Ex: $D_r(z_0) = |z - z_0| < r$ open $\quad \mathrm{Re}(z) \geq 0$ closed

$|z| \leq 2$ closed

$0 < |z - 1| \leq 1$ neither

$2 < \mathrm{Im}(z) < 3$ HW

$\mathbb{C}$ open + closed

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Panel 13

Definition: A set S is connected

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