Panel 1

Complex Analysis

HW #3

① We know that $\arg(zw) = \arg(z) + \arg(w)$. Is it true for Arg?
In other words: is $\mathrm{Arg}(zw) = \mathrm{Arg}(z) + \mathrm{Arg}(w)$ for all $z, w \in \mathbb{C}$?

② Use de Moivre's Formula to derive the following trig identities:
a) $\cos(3\theta) = \cos^3(\theta) - 3\cos\theta\sin^2(\theta)$
b) $\sin(3\theta) = 3\cos^2(\theta)\sin(\theta) - \sin^3(\theta)$

③ Take $z = -1+i$, $w = 2+i$. Show graphically
$|z|$, $\arg(z)$, z^2, $1/z$, $z \cdot w$, and $\overline{w}$. Confirm algebraically.

④ Suppose $\omega = e^{i\frac{2\pi}{5}}$. Draw $\omega^0, \omega^1, \omega^2, \omega^3, \omega^4$ and ω^5

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continued ⇒

Panel 2

⑤ Find both square roots of $(-i)$

⑥ Find the three cube roots of $(-8i)$

⑦ Visit **http://math.mit.edu/daimp/ComplexRoots.html**
Zoom in and click on $z=1$ (as best as you can).
Look at the $2^{nd}, 3^{rd}, 4^{th}, \ldots, 9^{th}$ roots and describe the pattern in words. Does the same pattern come out for the n-th roots of -1? How about for any $z \neq 0$?
Make up a theorem like: If z is any non-zero complex number, then we can say the following about the n-th roots of z: ______________

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