Panel 1

Last time:

$z \in \mathbb{C}$ as vectors : $z = x+iy = (x,y)$

$\|z\| = \sqrt{x^2+y^2}$ ⊛ $\boxed{|z|^2 = z\bar{z}}$ -

$\bar{z} = x - iy$ $\qquad \overline{(1-i)} = 1+i$

$\arg(z)$ is angle of z (in polar coords)

$\operatorname{Arg}(z)$ is principle angle $-\pi < \theta \leq \pi$

$e^{i\pi}+1=0$

Euler's Formula: $e^{i\theta} = \cos(\theta) + i\sin(\theta)$

$z = re^{i\theta} = r(\cos\theta + i\sin\theta)$

To multiply z, w : mult radii, add angles

1

Panel 2

$|z - z_0| = R$

$|z-z_0|^2 = R^2$

$(z-z_0)\overline{(z-z_0)} = (z-z_0)(\bar{z}-\bar{z}_0) =$

$= z\bar{z} - z\bar{z}_0 - z_0\bar{z} + z_0\bar{z}_0$

$= |z|^2 - (z\bar{z}_0 + \overline{z\bar{z}_0}) + |z_0|^2$

$= |z|^2 - 2\operatorname{Re}(z\bar{z}_0) + |z_0|^2 = R^2$

① $|z|^2 = z\bar{z}$

② $\frac{w+\bar{w}}{2} = \operatorname{Re}(w)$

$w + \bar{w} = 2\operatorname{Re}(w)$

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Panel 3

Note: $(e^{i\theta})^n = e^{in\theta}$

de Moivre's Formula: $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$

n=2: $(\cos(\theta) + i\sin\theta)^2 = \cos(2\theta) + i\sin(2\theta)$

$\cos^2\theta - \sin^2\theta + 2i\cos\theta\sin\theta = \cos(2\theta) + i\sin(2\theta)$

$\Rightarrow \cos^2\theta - \sin^2\theta = \cos(2\theta)$ double angle formulas

$2\cos\theta\sin\theta = \sin(2\theta)$

HW: triple angle formula

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Panel 4

Know: $(e^{i\theta})^n = e^{in\theta}$ $\forall n$ ~~≥0~~ integers

$[e^{i\theta}]^{-1} = \frac{1}{e^{i\theta}} = \frac{1}{\cos\theta + i\sin\theta} \cdot \frac{\cos\theta - i\sin\theta}{\cos\theta - i\sin\theta} =$

$= \frac{\cos\theta - i\sin\theta}{\cos^2\theta + \sin^2\theta} = \cos(\theta) - i\sin\theta =$

$= \cos(-\theta) + i\sin(-\theta) =$

$= e^{-i\theta}$

$(e^{i\theta})^{-1} = e^{-i\theta}$

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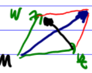
Panel 5

Visualize operations with complex numbers

$z+w$ } diags in parallelogram
$z-w$



$c\cdot z$, $c\in\mathbb{R}$ z just longer or shorter or inverted

$|z| = R$ circle

$\arg(z)$ = angle

$z\cdot w$ mult. radii, add angles

z/w div. radii, subtract angles

$\bar{z}$

z^n

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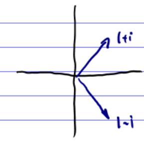
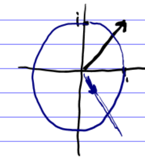
Panel 6

Ex: If $z = 1+i$, find $\bar{z}$, $1/z$ alg. and visually

$\bar{z} = 1-i$ $\frac{1}{z} = \frac{1}{1+i} = \frac{1-i}{2} =$





$\bar{z}$ reflect about x-axis

$\frac{1}{z}$ is refl. about x-axis, inversion about unit circle

$\frac{1}{z} = \frac{1}{z}\cdot\frac{\bar{z}}{\bar{z}} = \frac{1}{z\bar{z}}\cdot\bar{z} = \frac{1}{|z|^2}\bar{z}$

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Panel 7

Ex: Let $z = \frac{-2}{1+\sqrt{3}i}$, find $\arg(z)$

$\frac{-2}{1+\sqrt{3}i}\cdot\frac{1-\sqrt{3}i}{1-\sqrt{3}i} = \frac{-2(1-\sqrt{3}i)}{4} = \frac{1}{2}(-1+\sqrt{3}i)$

$\tan(\theta) = \sqrt{3}$, $\theta = \pi/3$

$\arg(z) = \frac{2\pi}{3}$

$\arg(z) = \arg\left(\frac{-2}{1+\sqrt{3}i}\right) = \arg(-2) - \arg(1+\sqrt{3}i)$
$= \pi - \pi/3 = 2\pi/3$

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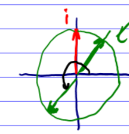
Panel 8

Complex Roots: geometrically

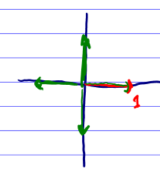
$\sqrt{i} = z$

or $z^2 = i$



$\sqrt[4]{1} = z$

$z^4 = 1$



$\sqrt[5]{1}$

$\sqrt[3]{1} =$

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Panel 9

Find the n-th roots of unity:

HW

http://math.mit.edu/daimp/ComplexRoots.html

Panel 10

http://math.mit.edu/daimp/ComplexRoots.html