Panel 1

Welcome to Math 4512

Complex Analysis

(where $z^2+1=0$ does have solutions
and numbers are no longer sorted)

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Panel 2

Math 4512 : Complex Analysis

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Grading:
2 exams (45%)
HW (45%)
Participation (10%)

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Panel 3

Other Stuff: $\boxed{\text{Dyknow}}$

Material covered: Chapters 1 to 6 + extras

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Panel 4

Dyknow: Note-taking and Collaboration Tool

- download the program
- double-click on downloaded software to install

You will be prompted for some information to enter. Use the following:

- for "*DyKnow Server Address*", use: `dyknow://vision.dyknow.com/shu.edu`

Leave everything else as it is. In addition, you will need a user name and password, which you will receive from your instructor.

User name: user name

Password: username

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Panel 5

Definition of Complex Numbers:

Pairs $z = (x, y)$ where x, y are real, and the sum and product defined as:

$z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$

$z_1 + z_2 = (x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$ ~ boring

$(1, 2) + (3, 4) =$

$z_1 \cdot z_2 = (x_1, y_1) \cdot (x_2, y_2) = (x_1 \cdot x_2 - y_1 \cdot y_2, x_1 y_2 + x_2 y_1)$

$(1, 2) \cdot (3, 4)$

$z \in (x, y)$, where $x = Re(z)$ real part

$y = Im(z)$ imaginary part

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Panel 6

$z_1 + z_2 = (x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$ - boring

$z_1 \cdot z_2 = (x_1, y_1) \cdot (x_2, y_2) = (x_1 x_2 - y_1 y_2, x_1 y_2 + x_2 y_1)$ ~ no sense

why not $(x_1, y_1) \cdot (x_2, y_2) = (x_1 x_2, y_1 y_2)$

Want: $z \cdot w = 0 \Rightarrow z = 0, w = 0 = (0, 0)$

But $(x, 0) \cdot (0, y) = (0, 0)$ for all x, y

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Panel 7

$z_1 + z_2 = (x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$

$z_1 \cdot z_2 = (x_1, y_1) \cdot (x_2, y_2) = (x_1 x_2 - y_1 y_2, x_1 y_2 + x_2 y_1)$

What if $z_1 = (x_1, 0)$ and $z_2 = (x_2, 0)$?

$\Rightarrow z_1 + z_2 = (x_1 + x_2, 0)$

and $z_1 \cdot z_2 = (x_1, 0) \cdot (x_2, 0) = (x_1 \cdot x_2 - 0, 0 + 0) = (x_1 \cdot x_2, 0)$

Thus: $\mathbb{R} \subset \mathbb{C}$ = complex numbers (with $\mathrm{Im}(z) = 0$)

Recall: $\mathbb{N} \subset \mathbb{Q}$

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Panel 8

<u>What's the Deal (I mean, really)?</u>

Take $z_1 = (1, 2)$ and $z_2 = (3, 4)$, find $z_1 \cdot z_2$.

$(1, 2) \cdot (3, 4) = (3 - 8, 4 + 6) = (-5, 10)$

Not helpful

Take $z = (0, 1)$, find $z^2 = z \cdot z$

$(0, 1) \cdot (0, 1) = (0 - 1, 0 + 0) = \underline{(-1, 0)}$

Solved $z^2 = -1$ $\quad z^2 = -4 \Rightarrow z = (0, 2)$ ✓

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Panel 9

There is a (complex) number $z = (0,1) = i$ with $z^2 = -1$!

Compute: $\underbrace{(x,0)} + \underbrace{(0,1)} \cdot \underbrace{(y,0)}$

$(x,0) + (0 \cdot 0, 0 + y) = (x,0) + (0,y) = \underline{(x,y)}$

<u>Definition:</u> $i = (0,1)$ has property $i^2 = -1$

<u>Theorem:</u> Every complex number $z = (x,y)$ can be written as

$$z = x + iy$$

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Panel 10

Now everything makes sense:

<u>Ex:</u> For $z_1 = (1,2)$ and $z_2 = (3,4)$, find $z_1 \cdot z_2$:

$z_1 = 1 + 2i$, $z_2 = 3 + 4i$

$z_1 \cdot z_2 = (1+2i)(3+4i) = 3 + 4i + 6i + \underline{8i^2}$

$= 3 + i(4+6) - 8 = -5 + 10i = \underline{(-5,10)}$

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Panel 11

Ex: Find $Re(z)$ and $Im(z)$ if

a) $z = i$ $\quad Re(z) = 0$

$Im(z) = 1$

$z = 3+4i \quad , Re(z) = 3 , \quad Im(z) = 4$

b) $z = (1+i)^2 =$

$= 1^2 + 2\cdot 1 \cdot i + i^2$

$= 2i$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a+b)(a-b) = a^2 - b^2$

$Re(z) = 0$

$Im(z) = 2$

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Panel 12

Basic Properties

All standard rules apply — associative, distrib., commut.

Ex: Find i^2, i^3, i^4, i^5, and i^{109}

i

$i^2 = -1$

$i^3 = -1 \cdot i = -i$

$i^4 = 1$

$i^5 = i$ $\quad i^6 = -1, i^7 = -i, i^8 = 1$

$i^{109} = ?$

a) 1

b) i

c) $-i$

d) -1

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Panel 13

Prove that $z_1 \cdot z_2 = z_2 \cdot z_1$

$$(x_1, y_1) \cdot (x_2, y_2) = (x_1 x_2 - y_1 y_2, x_1 y_2 + x_2 y_1)$$

$$(x_2, y_2) \cdot (x_1, y_1) = (x_2 x_1 - y_2 y_1, x_2 y_1 + y_2 x_1)$$

Prove Distributive law: $z_1 \cdot (z_2 + z_3) = z_1 z_2 + z_1 z_3$ HW

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Panel 14

Solve $5z + 9 = 1$ and $3z = 1$

$5z = -8$

$\underline{z = -8/5}$

$z = 1/3 + 0i$

Solve $(1+2i)z = 1$ $z = 1/(1+2i)$

$(1+2i)(x+iy) = 1$

$x - 2y + i(y + 2x) = 1 + i0$

$x - 2y = 1$ $x + 4x = 1 \Rightarrow x = 1/5, \; y = -2/5$

$2x + y = 0 \Rightarrow y = -2x$

Or: $z = \frac{1}{(1+2i)} \frac{(1-2i)}{(1-2i)} = \frac{1-2i}{1+4} = \frac{1-2i}{5} = \underline{\frac{1}{5} - \frac{2}{5} i}$

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Panel 15

$1/i \cdot \frac{i}{i} = \frac{i}{-1} = -i$

Try these:

a) $\mathrm{Im}(iz)$ and $\mathrm{Re}(1/i) = 0$ b) $(x+iy)^2 = x^2 - y^2 + 2ixy$

$\mathrm{Im}(i(x+iy)) = \mathrm{Im}(ix - y)$

b) Show that $1+i$ solves $z^2 - 2z + 2 = 0$ HW

c) Find $\mathrm{Re}\left(\frac{1+2i}{3+4i}\right)$ and $\mathrm{Im}\left(\frac{1+2i}{3+4i}\right)$

d) Solve $z^2 = i$, i.e. $\sqrt{i}$

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Panel 16

$$\frac{(1+2i)}{(3+4i)} \cdot \frac{(3-4i)}{(3-4i)} = \frac{3 - 4i + 6i - 8i^2}{25} = \frac{11 + i2}{25}$$

$$\sqrt{i} = \begin{cases} 1/\sqrt{2} + i\, 1/\sqrt{2} \\ -1/\sqrt{2} - i\, 1/\sqrt{2} \end{cases}$$

$\sqrt{i}$

$-1 \quad \sqrt{-1} = i, \quad \sqrt{i} = \sqrt{\sqrt{-1}} = (-1)^{1/4}$

$(-i)^4 = (-1)^4 (i)^4 = 1 \cdot 1$

$z^2 = i \Leftrightarrow (x+iy)^2 = i \Leftrightarrow x^2 - y^2 + 2xyi = i$

$x^2 - y^2 = 0 \Leftrightarrow x = \pm y$

$z = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$

$2xy = 1 \Rightarrow \pm 2xx = 1 \Rightarrow x = \pm 1/\sqrt{2}$

$z = -\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$

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