Panel 9

Homework

1. Does $\lim_{n \to \infty} \left(\frac{1+i}{\sqrt{2}} \right)^n$ exist?

2. Show convergence and evaluate:

   a) $\sum_{n=0}^{\infty} \frac{(1+i)^n}{2^n}$ b) $\sum_{n=1}^{\infty} \left(\frac{1}{2+i} \right)^n$

3. For which z does $\sum_{n=0}^{\infty} \frac{(z+i)^n}{2^n}$ converge?

4. Use the ratio test to check convergence for:

   a) $\sum_{n=1}^{\infty} \frac{(1+i)^n}{n 2^n}$ b) $\sum_{n=1}^{\infty} \frac{(1+i)^n}{n!}$ c) $\sum_{n=0}^{\infty} \frac{(1+i)^{2n}}{(2n+1)!}$

5. Does $\sum_{n=0}^{\infty} \frac{(3+4i)^n}{5^n n^2}$ converge? (use comp. thm. with p-series)

$\Rightarrow$

Panel 10

(6) For $|z| < 1$ define $f(z) = \sum_{n=0}^{\infty} z^{(2^n)} = z + z^2 + z^4 + z^8 + \dots$

Show that $f(z) = z + f(z^2)$

(7) Not HW, just for curiosity: if $z = r\,\mathrm{cis}(\theta)$ with $r < 1$

show that

$$\sum_{n=0}^{\infty} z^n = \sum_{n=0}^{\infty} r^n \mathrm{cis}(n\theta) = \frac{1 - r\cos(\theta) + i r \sin(\theta)}{1 + r^2 - 2r\cos(\theta)}$$

Conclude from that:

$$\sum_{n=0}^{\infty} r^n \cos(n\theta) = \frac{1 - r\cos(\theta)}{1 + r^2 - 2r\cos(\theta)}$$

$$\sum_{n=0}^{\infty} r^n \sin(n\theta) = \frac{r\sin(\theta)}{1 + r^2 - 2r\cos(\theta)}$$