

Panel 6

## Homework

- ① A function  $u(x, y)$  is called **harmonic** if  $\frac{\partial^2}{\partial x^2} u + \frac{\partial^2}{\partial y^2} u = 0$
- Show that  $u(x, y) = e^x \cos(y)$  and  $v(x, y) = e^x \sin(y)$  are harmonic.
- ② Are  $u(x, y) = x^2 - y^2$  and  $v(x, y) = x^3 - 3xy^2$  harmonic?
- What about  $u(x, y) \cdot v(x, y)$ ?
- ③ For the following functions  $u(x, y)$  find  $v(x, y)$  such that  $f(z) = u + iv$  is analytic:
- a)  $u(x, y) = y^3 - 3x^2y$     b)  $u(x, y) = e^y \sin(x)$
- c)  $u(x, y) = \sin(y) \sinh(y)$
- ④ Suppose  $f(z) = u(x, y) + iv(x, y)$  is analytic and  $u_x, u_y, v_x, v_y$  are cont. Show that  $u$  and  $v$  are harmonic.