

Panel 12

Homework:

① Prove that the image of a disk under a linear transformation is a disk.

② Find the image under the map  $f(z) = z^2$  of

a) vertical line  $\{(x, y) : x = 2\}$

b) infinite strip  $\{(x, y) : 1 < x < 2\}$

③ Find the following limits:

a)  $\lim_{z \rightarrow 2+i} (z^2 - 4z + 2 + 5i)$

b)  $\lim_{z \rightarrow i} \frac{z^4 - 1}{z - i}$

④ next page ...

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$$(4) \text{ Let } f(z) = \frac{z^2}{|z|^2} = \frac{x^2 - y^2 + 2ixy}{x^2 + y^2}$$

a) Find  $\lim_{z \rightarrow 0} f(z)$  as  $z \rightarrow 0$  along  $y = x$

b) Find  $\lim_{z \rightarrow 0} f(z)$  as  $z \rightarrow 0$  along  $y = 2x$

c) Find  $\lim_{z \rightarrow 0} f(z)$  as  $z \rightarrow 0$  along  $y = x^2$

What can you conclude about  $\lim_{z \rightarrow 0} f(z)$  ?

(5) Let  $f(z) = \frac{\bar{z}}{z}$ . Does  $f(z)$  have a limit as  $z \rightarrow 0$  ?