MATH 3912 - Definitions and Theorems (2)

Definitions

1. The determinant of an $n \times n$ matrix

2. A linear vector space X

3. Vectors v_1, v_2, ..., v_n are linearly dependent (or independent)

4. The space $L^p([a,b])$, where $p > 0$ and $[a,b]$ is an interval in $\boldsymbol{R}$

5. A function $f : [a,b] \to \boldsymbol{R}$ is bounded

6. A function $f : [a,b] \to \boldsymbol{R}$ is continuous

7. A function $f : [a,b] \to \boldsymbol{R}$ is uniformly continous

8. A function $f : [a,b] \to \boldsymbol{R}$ is Lipschitz with power α, i.e. $f \in Lip(\alpha)$

9. The space $C^n([a,b])$, where $[a,b]$ is an interval in $\boldsymbol{R}$

10. The space $C^\infty([a,b])$, where $[a,b]$ is an interval in $\boldsymbol{R}$

11. A Taylor series for a function f centered at a point c

12. A power series centered at a point c

13. A function that is real analytic

14. A function of a complex variable that is analytic

15. What is an entire function?

16. What is a polynomial?

17. What is a functional, and what is a linear functional?

18. What is a conformal map?

Theorems

1. When does a system of linear equations $\Sigma_{j=1}^{n} a_{ij} x_j = b_j$ for $i = 1, 2, \ldots, n$ possess a unique solution?

2. What conditions can you put on a function f and its domain to ensure it is bounded?

3. What conditions can you put on a function f and its domain to ensure it is uniformly continous?

4. What is the *Mean Value Theorem* for a function $f : [a,b] \to R$?

5. What is the *First Mean Value Theorem for Integral*?

6. What is *Rolle's Theorem*?

7. What is the *Generalized Rolle's Theorem*?

8. What is Taylor's Theorem?

9. What is *Cauchy's Integral Formula*?

10. What is the *Fundamental Theorem of Algebra* (and, by the way, what is the Fundamental Theorem of Calculus?)

11. What is the *Factorization Theorem* and the *Uniqueness Theorem*?

12. What is the image of circles, radius $r \geq 1$, under the (complex) function $f(z) = \frac{1}{2}\left(z + \frac{1}{z}\right)$?