MATH 3912 - Definitions and Theorems

Below are terms that we defined in various presentations. Please look up the definition in the papers (see our website) or in the copy of the introductory chapter of the text book.

1. The determinant of an $n \times n$ matrix
2. A linear vector space X
3. Vectors v_1, v_2, ..., v_n are linearly dependent (or independent)
4. The space $L^p([a,b])$, where $p > 0$ and $[a,b]$ is an interval in $\boldsymbol{R}$
5. A function $f : [a,b] \to \boldsymbol{R}$ is bounded
6. A function $f : [a,b] \to \boldsymbol{R}$ is continuous
7. A function $f : [a,b] \to \boldsymbol{R}$ is uniformly continous
8. A function $f : [a,b] \to \boldsymbol{R}$ is Lipschitz with power α, i.e. $f \in Lip(\alpha)$
9. The space $C^n([a,b])$, where $[a,b]$ is an interval in $\boldsymbol{R}$
10. The space $C^\infty([a,b])$, where $[a,b]$ is an interval in $\boldsymbol{R}$
11. A Taylor series for a function f centered at a point c
12. A power series centered at a point c
13. A function that is real analytic

Next are some questions that can be answered by stating a theorem we encountered in our introduction. Please answer the questions and memorize the corresponding theorem; you might want to try to paraphrase the theorems in your own words, or try to interpret them geometrically.

1. When does a system of linear equations $\Sigma_{j=1}^n a_{ij} x_j = b_j$ for $i = 1, 2, \ldots, n$ possess a unique solution?
2. What conditions can you put on a function f and its domain to ensure it is bounded?
3. What conditions can you put on a function f and its domain to ensure it is uniformly continous?
4. What is the *Mean Value Theorem* for a function $f : [a,b] \to R$?
5. What is the *First Mean Value Theorem for Integral*?
6. What is *Rolle's Theorem*?
7. What is the *Generalized Rolle's Theorem*?
8. What is Taylor's Theorem?