MATH 3912 - Assignment 4

1. Suppose $f(x) = x(x^2 - 1)$, defined on the interval $(-1, 1)$. Does Rolle's theorem apply? If so, find the point $c \in (-1, 1)$ that is guaranteed to exist (there could be more than one correct answer).

2. If $f(x) = x(x^2 - 1)$ on $(-1, 1)$ then $f(-1) = f(0) = f(1) = 0$, so the Generalized Rolle's theorem applies. Find all points that are guaranteed to exist by the generalized Rolle's theorem in this case. *Hint:* There should be two points where $f' = 0$ and one point where $f'' = 0$.

3. Show that $f(x) = x^{\frac{4}{3}}$ is $C^1(\mathbf{R})$ but not $C^2(\mathbf{R})$. What about $f(x) = x^{\frac{7}{3}}$?

4. Find a function $f(x)$ that is C^n but not C^{n+1} on the real line (you can not use the function below even though it does have the right properties, but you could make up a function that's similar to the ones *above*).

5. Show that the function f defined below is $C^{k-1}(\mathbf{R})$ but not $C^k(\mathbf{R})$ on $(-1, 1)$.

$$f(x) = \begin{cases} x^k & \text{for} \quad x \geq 0 \\ 0 & \text{for} \quad x < 0 \end{cases}$$

6. Show that the function f defined below is continuous but not differentiable at $x = 0$ (which would put this function into $C^?$)

$$f(x) = \begin{cases} x sin(\frac{1}{x}) & \text{for} \quad x \neq 0 \\ 0 & \text{for} \quad x = 0 \end{cases}$$

7. Find the 6-th Taylor polynomial for the function $f(x) = \cos(x)$ around $x = 0$. Then use that polynomial to approximate the value of $\cos(0.5)$. Use a calculator to compare your approximation with the actual value of $\cos(0.5)$ (don't forget to switch your calculator into radian).

8. Find the 5-th Taylor polynomial for the function $f(x) = \ln x$ around $x = 1$ and use it to approximate the value of $\ln(1.5)$. Use a calculator to compare your approximation with the actual value.

9. Find the 3-rd Taylor polynomial for the function $f(x) = x(x^2 - 1)$ around $x = -1$. Then find the 10-th and the 20-th Taylor polynomial for that function around $x = -1$.

10. Show that if $f(x)$ is a polynomial then $\lim_{x \mapsto \infty} f^{(n)}(x) = 0$ for all x. Is the converse also true?

11. For many functions such as e^x and $\sin(x)$ or $\cos(x)$ we can use higher and higher Taylor polynomials to get better and better approximation (polynomials of higher and higher degrees). Does this process work for every function?