MATH 3912 - Assignment 2

1. Recall that a function $f : D \mapsto R$ is bounded if there exists a constant M such that $|f(x)| \leq M$ for all $x \in D$. For our discussion assume that D is an interval such as (a, b), $[a, b]$, $(-\infty, b)$, (a, ∞), or corresponding half-open intervals on the real line.

   Find a function that is not bounded. Is it true that every continuous function is bounded? If not, find conditions on the domain that ensure that continuous functions *are* bounded. Are all differentiable functions bounded?

2. For what values of a and b are the functions $f(x) = ax^b$ bounded on $[0, 1]$?

3. For what values of a and b are the functions $f(x) = \frac{1}{x^2+ax+b}$ bounded on $[-1, 1]$.

4. Consider the function $f(x) = 1/x^2$. Is the function:
   (a) Continuous on the interval $[0, 1]$?
   (b) Continuous on the interval $(0, 1)$?
   (c) Continuous on the interval $[1, 2]$?
   (d) Continuous on the interval $[1, \infty)$?
   (e) Uniformly continous on the interval $[0, 1]$?
   (f) Uniformly continous on the interval $(0, 1)$?
   (g) Uniformly continuous on the interval $[1, 2]$?
   (h) Uniformly continuous on the interval $[1, \infty)$?

5. Show that the function $f(x) = e^{-x}$ is uniformly continuous over the interval $[0, \infty)$. What about on the interval $(-\infty, 0)$ (explain by means of a picture)?

6. Recall that a function f is $L^p[a, b]$ if $\int_a^b |f(x)|^p dx$ exists.
   (a) Is the function $f(x) = \frac{1}{x} \in L^1[0, 1]$?
   (b) Is the function $f(x) = \frac{1}{x^{1/2}} \in L^1[0, 1]$?
   (c) Is the function $f(x) = \frac{1}{x^{1/2}} \in L^2[0, 1]$?
   (d) For a fixed $0 \leq \alpha < 1$ find p such that $f(x) = \frac{1}{x^\alpha} \in L^p[0, 1]$?