MATH 3912 - Assignment 1

1. Compute the determinant of

$$\begin{pmatrix} 6 & 0 & -8 \\ 8 & 3 & 0 \\ 6 & 3 & -4 \end{pmatrix}$$

2. Compute the determinant of the lower triangular matrix

$$\begin{pmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$$

3. Suppose A is a lower triangular n by n matrix as defined below. Show that $\det(A) = \prod_{i=1}^{n} a_{ii}$, or in other words the determinant of A is the product of the entries in the main diagonal.

$$A = \begin{pmatrix} a_{11} & 0 & \dots & & & 0 \\ a_{21} & a_{22} & 0 & \dots & & 0 \\ a_{31} & a_{32} & a_{33} & 0 & \dots & 0 \\ \vdots & & & \ddots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & & a_{nn} \end{pmatrix}$$

4. Find the determinant of the matrix V defined as follows:

$$V = \begin{pmatrix} 1 & z_0 & z_0^2 \\ 1 & z_1 & z_1^2 \\ 1 & z_2 & z_2^2 \end{pmatrix}$$

5. Solve the following system of equations, if there is a solution:

$$\begin{aligned} 8x_1 - 2x_2 + x_3 &= 1 \\ 2x_1 - 8x_2 &= 7 \\ -2x_1 - 6x_2 - 7x_3 &= 2 \end{aligned}$$

6. Suppose $p(x) = a_0 + a_1 x + a_2 x^2$ is a quadratic polynomial. Find the coefficients of that polynomial so that

$$\begin{aligned} p(-1) &= 1 \\ p(0) &= 4 \\ p(1) &= 1 \end{aligned}$$

7. Suppose

$$\vec{v}_1 = \langle 7, -5, 5 \rangle, \ \vec{v}_2 = \langle 6, 0, 0 \rangle, \text{ and } \vec{v}_3 = \langle -3, 5, 0 \rangle$$

are three vector in R^3. Are they linearily independent? What about the vectors

$$\vec{w}_1 = \langle -3, -6, -7 \rangle, \ \vec{w}_2 = \langle 0, 2, -15 \rangle, \text{ and } \vec{w}_3 = \langle 3, 8, -8 \rangle$$

How about

$$\vec{z}_1 = \langle 6, 0, 0 \rangle, \ \vec{z}_2 = \langle -3, 5, 0 \rangle, \ \vec{z}_3 = \langle -3, -6, -7 \rangle, \text{ and } \vec{z}_4 = \langle 0, 2, -15 \rangle$$